Inspection Based Probabilistic Modeling of Fatigue Crack Progression

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Abstract: This paper focuses on one of methods which can be used for reliability assessment of structures which are subject to cyclic loads. A particular attention is paid to creation and propagation of fatigue cracks from edges and surface. On the basis of the reliability assessment, a system of inspections is proposed for structural details which tend to be sensitive to fatigue damage. A new method which is still under development - Direct Optimized Probabilistic Calculation ("DOProC") – was used for this probabilistic task. This method is the basis of the FCProbCalc software.

Key-Words: DOProC, Direct Optimized Probabilistic Calculation, FCProbCalc, probabilistic methods, fatigue crack, inspection of structure, reliability assessment, random variable, probability of failure

1 Introduction
This paper describes the use of the original method and the method which is under development now: the Direct Optimized Probabilistic Calculation ("DOProC") uses a purely numerical approach without any simulation techniques. This provides more accurate solutions to the probabilistic tasks, and, in some cases, results in considerably faster completion of computations. Such solution entails a small numerical error only and minor inaccuracies, the reason being discretization of input and output quantities (for instance,[2]).

Probabilistic and stochastic methods are used in engineering tasks. The advantage is, in particular, that the calculation [8] or reliability assessment of building constructions takes into account the random nature of input quantities.

These approaches need a rather big database of input quantities obtained by modeling[6, 7], laboratory measurements [15] or directly in site [12, 14].

It was decided to use DOProC to model the probability of propagation of fatigue cracks in a construction which is subject to load cycles. Theoretical backgrounds were described in detail in[4, 5]. DOProC can be used now to solve efficiently a number of probabilistic computations.

DOProC has proved to be a good solution, for instance, in probabilistic analyses of fatigue crack propagation in constructions subject to cyclical loads. Detailed methods with examples of the probabilistic assessment for a construction subject to fatigue load are available, a particular attention being paid to cracks from the edge and those from surface. Similarly to other probabilistic analysis [3], this information is used as a basis for proposing a system of inspections of the construction. In order to improve quality of probabilistic calculations, special software – FCProbCalc [9]– was developed. Using this software, the task can be solved flexibly in a user-friendly environment.

2 Formulating the probability task
Reliability of the bearing structure has been significantly influenced by degradation resulting, in particular, from the fatigue of the basic materials[17]. When investigating into the propagation, the fatigue crack that deteriorates a certain area of the structure components is described with one dimension only – a.

In order to describe the propagation of the crack, the linear elastic fracture mechanics [1] is typically applied. This method uses Paris-Erdogan’s law [13] and defines relation between propagation rate of the crack and swing of the stress rate coefficient, \( \Delta K \), in the face of the crack:

\[
\frac{da}{dN} = C(\Delta K)^m ,
\]

where \( C, m \) are material constants, \( a \) is the crack size and \( N \) is the number of loading cycles.

The fatigue crack will propagate in a stable way only if the initial crack \( a_0 \) exists in the place where...
the stress is concentrated. This place is located at the edge or on the surface of the element.

The primary assumption is that the primary design should take into account the effects of the extreme loading and the fatigue resistance should be assessed then. This means, the reliability margin in the probability task is:

\[ RF_{(R,S)} = R - S , \]  

(2)

where \( R \) is the random resistance of the element and \( S \) is the random variable effect of the extreme load. If such element is subject to the operating load, following cases can occur:

a) **safe service life** – the fatigue effects do not degrade the element by means of the fatigue crack,

b) **acceptable failure rate** – the fatigue effects degrade the element and decrease the load-bearing capacity of the element,

c) **acceptable failure rate** - fatigue effects are expressed as stress changes.

The calculation model of the fatigue crack propagation defines the stress when the maximum acceptable crack results in the constant resistance of the structure, \( R \), that corresponds to the stress in the yield point \( f_y \). The approach c) is more demonstrative and has been preferred to the approach b) because it describes the non-linear growth of the both stresses in the element under degradation.

When using (1), the condition for the acceptable crack length, \( a_{ac} \), is:

\[ N = \frac{1}{C} \int_{a_0}^{a_{ac}} \frac{da}{C(\Delta K)^m} > N_{tot} , \]  

(3)

where \( N \) is the number of cycles needed to increase the crack from the initiation size \( a_0 \) to the acceptable crack size \( a_{ac} \), and \( N_{tot} \) is the number of cycles throughout the service life.

The equation for the propagation of the crack size (1) needs to be modified for this purpose. The swing of the stress rate coefficient, \( \Delta K \), at the constant stress swing, \( \Delta \sigma \), is:

\[ \Delta K = \Delta \sigma \sqrt{\pi a} F_{(a)} , \]  

(4)

where \( F_{(a)} \) is the calibration function which represents propagation of the crack (for instance, from the edge or surface). After the change of the number of cycles from \( N_1 \) to \( N_2 \), the crack will propagate from the length \( a_1 \) to \( a_2 \). Having modified (1) and using (4), the following formula will be achieved:

\[ \int_{a_1}^{a_2} \frac{da}{\sqrt{\pi a} F_{(a)}} = \int_{N_1}^{N_2} C(\Delta \sigma)^m dN . \]  

(5)

If the length of the crack \( a_1 \) equals to the initial length \( a_0 \) (this is the assumed size of the initiation crack in the probabilistic approach) and if \( a_2 \) equals to the final acceptable crack length \( a_{ac} \) (this is the acceptable crack size which replaces the critical crack size \( a_c \) if the crack results in a brittle fracture - but in order to calculate the phenomenon (10) \( a_2 \) can be equal to the size of the detectable crack, \( a_d \)), then the left-hand side of the equation (5) can be regarded as the resistance of the structure - \( R \):

\[ R_{(a_d)} = \int_{a_1}^{a_d} \frac{da}{\sqrt{\pi a} F_{(a)}} . \]  

(6)

Similarly, it is possible to define the cumulated effect of loads that is equal to the right side (random variable effects of the extreme load) (5):

\[ S = \int_{N_0}^{N} C(\Delta \sigma)^m dN = C(\Delta \sigma)^m(N_0 - N) , \]  

(7)

where \( N \) is the total number of stress peak swings, \( \Delta \sigma \), when the crack size increases from \( a_0 \) to \( a_{ac} \), and \( N_0 \) represents the number of swings in the time of the fatigue crack initiation (it is typically equal to zero).

It is possible to define a failure probability function \( RF \):

\[ RF_{(x)} = R_{(a_d)} - S_{(x)} , \]  

(8)

where \( X \) is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack.

The failure probability \( p_f \) equals to:

\[ p_f = P(RF_{(x)} < 0) = P(R_{(a_d)} - S_{(x)} < 0) . \]  

(9)

### 3 Inspection times

Because it is not certain in the probabilistic calculation whether the initial crack exists and what the initial crack size is and because other inaccuracies influence the modeling of the crack propagation, a specialized inspection is necessary to check the size of the detectable crack in a specific period of time. The factor which influences most the time of inspection is the acceptable size of the crack.

While the fatigue crack (see Fig. 1) is propagating, it is possible to define three random phenomena that are related to the growth of the
fatigue crack and may occur in any time, \( t \), during the service life of the structure. Then:

- **\( U(0) \) phenomenon**: No fatigue crack failure has not been revealed within the \( t \) time and the fatigue crack size \( a(t) \) has not reached the detectable crack size, \( a_d \). This means:
  \[ a(t) < a_d \tag{10} \]

- **\( D(0) \) phenomenon**: A fatigue crack failure has been revealed within the \( t \) time and the fatigue crack size \( a(t) \) is still below the acceptable crack size \( a_{ac} \). This means:
  \[ a_d \leq a(t) < a_{ac} \tag{11} \]

- **\( F(0) \) phenomenon**: A failure has been revealed within the \( t \) time and the fatigue crack size \( a(t) \) has reached the acceptable crack size \( a_{ac} \). This means:
  \[ a(t) \geq a_{ac} \tag{12} \]

\[ Fig. 1: \text{Probabilistic growth of the fatigue crack in the course of time} \]

Using the phenomena above, it is possible to define probability for their occurrence in any \( t \) time. Those three phenomena cover the complete spectrum of phenomena that might occur in the \( t \) time. This means:
\[ P(U(0)) + P(D(0)) + P(F(0)) = 1 . \tag{13} \]

The probabilistic calculation is carried out in time steps where one step typically equals to one year of the service life of the construction. When the probability of failure \( P(F(0)) \) reaches the designed failure probability \( P_{df} \), an inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection in the \( t \) time may result in any of the three mentioned phenomena. The inspection provides information about conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations.

If no fatigue cracks are found, the analysis of inspection results give conditional probability during occurrence. Using the inspection results for the \( t \) time, it is possible to define the probability of the mentioned phenomena in another times: \( T > t \).

For that purpose, the conditional probability should be taken into consideration. In order to determine the time for the next inspection, it is necessary to define the conditional probabilities \( P(F(T|x_U(0))) \) and \( P(F(T|x_D(0))) \), which can be expressed using the full probability law (details – see, for instance,[11]).

If re-distribution of stress from a point that is weakened by the crack is not taken into account, the crack propagation crack is usually rather high in the practical range of detectable values. If a fatigue crack is found during the inspection, it is necessary to monitor the safe growth of the crack or to take actions that will slow down or stop further propagation of the fatigue crack.

Those approaches which are based on the calculated DOProC probability of three basic phenomena, (10) to (12), using the reliability function, \( RF \), for each year of operation of the construction were included into FCProbCalc. Using this software application, it is possible to monitor effectively and flexibly development of fatigue damage in steel structures, to determine times for inspections and to ensure that the construction will be fit for operation in terms of fatigue safety.

4 DOProC probabilistic calculation

FCProbCalc has been developed using the aforementioned techniques. By means of FCProbCalc ("Fatigue Crack Probability Calculation"), it is possible to carry out the probabilistic modeling of propagation of fatigue cracks in a user friendly environment and to propose a system of regular inspections which should reveal damage to the structure.

\[ Fig. 2: \text{FCProbCalc desktop with description of all input quantities which were entered into the system} \]

The reference probabilistic calculation in FCProbCalc included the probabilistic assessment of a steel/reinforced concrete bridge fromon the
highway in a point where a longitudinal beam connects to a transversal beam. (For more details see [10]). The input quantities were determined deterministically or stochastically using non-parametric (empiric) and parametric probability distributions (see Fig. 2). The required reliability was described by the reliability index $\beta = 2$ which corresponded to the designed probability of failure $P_d = 0.02277$.

The probabilistic calculation was carried out for fatigue cracks propagating from the edge and surface. If a period of time is specified and the step is 1 year, it is possible to determine load effects, $S$, pursuant to (7) – see Fig. 3, resistance of the construction $R_{(ad)}$ and $R_{(aac)}$ pursuant to (6) – see Fig. 4 (so far, five types of numerical integration are available) as well as the probability of elemental phenomena, $U$, $D$ and $F$, pursuant to (10) through (12) which are the basis for specification of inspection times.

Similarly, it is possible to show the resulting probability of the elemental phenomena (10) through (12) for the probabilistic modeling of propagation of a fatigue crack from the surface (Gaussian quadrature was chosen as a numerical integration method).

Fig. 5 shows results of the probabilistic modeling of a fatigue crack from the edge. The curves describe dependence of the probability of failure, $P_f$, on time of operation of the bridge construction. When the probability of failure exceeds the specified designed probability, $P_d$, the inspection should be performed. It was decided that the first inspection of the bridge should take place after 49 years of operation. This inspection will focus on growth of the fatigue crack on the edge.

Fig. 6 shows the resulting times for the proposed inspections of the construction, a
particular attention being paid to the growth of the fatigue crack from the surface.

Fig. 7: Dependence of the failure probability, $P_f$, on years of operation of the bridge construction during the probabilistic calculation of propagation of fatigue cracks from the surface (0 to 150 years) with respect to the conditional probability and specification of the time for the first and subsequent inspections of the bridge structure (Gaussian quadrature was chosen as a numerical integration method)

It follows from the comparison of times for the first inspections which focus on the fatigue damage by the both types of the fatigue cracks (after 49 years of operation for the edge crack and after 109 years of operation for the surface crack) that the fatigue cracks propagate from the surface with a considerably lower speed that the fatigue cracks which initiate at the edge.

4 Conclusion

This paper discusses development of the DOProC probabilistic method and its use in the reliability assessment of the constructions. A particular attention is paid to the theory and practical aspects of the probabilistic assessment of the constructions which are subject to fatigue and tend to create fatigue cracks. The result of this method is similar to other probabilistic approaches: proposal of a system of regular inspections of the construction.

Those computations were applied in FCPProbCalc which was used for the mathematical modeling of propagation of fatigue cracks from the edge and surface. A probabilistic reliability assessment of the constructions was also performed in this software – it was based on the exact definition of the permissible size of the fatigue crack. The probabilities were obtained for three basic phenomena which are related to propagation of the fatigue cracks. On the basis of those data, the probability of failure can be calculated for each year of operation of the construction. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the construction which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined.

The methods and application can considerably improve estimation of maintenance costs for the structures and bridges subject to cyclical loads.

If this methodology is developed further, the goal of investigations seems to be, in particular, application of Bayesian networks [16] in the computational model which describes propagation of fatigue cracks.

Appendix

For a lite version of FCPProbCalc and other software products based on DOProC visit http://www.fast.vsb.cz/popv.

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