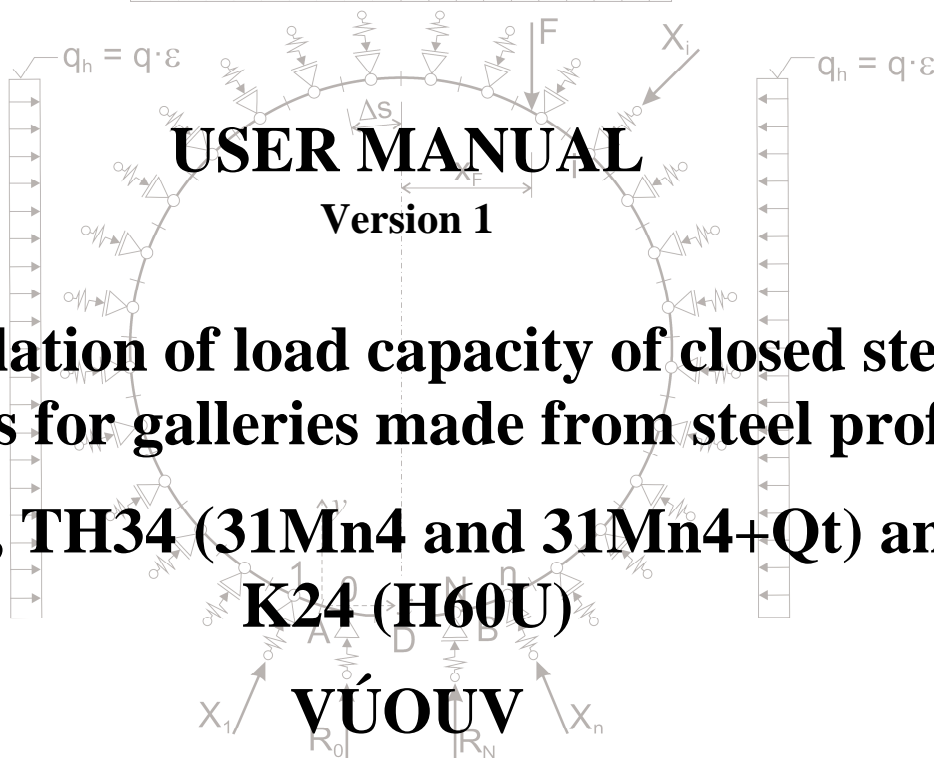


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**Faculty of Civil Engineering**

**Department of Structural Mechanics**



**Calculation of load capacity of closed steel supports for galleries made from steel profiles**

**TH29, TH34 (31Mn4 and 31Mn4+Qt) and K24 (H60U)**

**VÚOÚV**

Prepared for  
the Premegovnik Mine (Velenje) in Slovenia

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## 1. Introduction

This User Manual is intended for those who use the software named Calculation of load capacity of closed steel supports for galleries made from TH29, TH34 and K24 profiles – VÚOOV. TH29 and TH34 are made from 31Mn4 or 31Mn4+QT. K24 is made from H60U. In this section, corrosion is likely to reduce equally the cross-section by 10%, 20% and 30%.

The software calculates the load capacity of yielding and non-yielding steel supports made from the specified sections. It uses the 1<sup>st</sup> order theory and is based on assumption that the support is loaded with an active continuous load applied from top and sides on the mining or underground work.

The load capacity of the support is designed pursuant to ČSN EN 1993-1-1 (Eurocode 3: Design of steel structures). An alternative design uses rather high ultimate moments corresponding to the specified steel section and steel grade. It is possible to change the side/vertical loads. The support may consist of 3 up to 15 segments with different lengths and different radii of curvature. The slippage resistance of the connected segments ranges from 100 to 350 kN, the step being 50 kN. The software allows to calculate the load capacity of the symmetrically loaded symmetric supports (according to the vertical axis) as a function which depends on the side/vertical load ratio ranging from 0 to 2 with the step from 0.1 to 1.

In addition, it is possible (in order to simulate the load caused by the suspension groove) to load the support with a single force  $F$  applied in the specified place in the support level and an active continuous load applied from top and sides of the mining or underground work. In case of combined loads, the load capacity is always determined only for the specified side/vertical load ratio.

The User Manual is divided into two parts. The first part provides theoretical analyses which were the basis for development of the software application. Considering the user comfort, much attention was paid to the geometrical analysis of the closed steel supports. The purpose was to find a very simple way for entering the various shapes of the supports which differed, in fact, in the number, length and radius of curvature of individual elements. The static solution is based on a standard force method where each necessary integration is solved numerically. Components of the resultant line of the internal forces for the specified load determine places which limit the load capacity of the supports once the normal forces and plastic bending moments reach their ultimate values which were determined for the TH29 / TH34 / K24 sections made from the specified steel grades. The basis was the cross-section of the steel sections as well as knowledge of mechanical properties of the steel, available knowledge of bending tests, and, last but not least, the knowledge in bending test results. The sections were classified as the 1<sup>st</sup> class sections. This means, the load capacity can be calculated if plastic properties of the steel are assumed. Properties of the corroded support made from K24 were deduced theoretically,

without validation. The properties are an educated guess only, unless experiments are carried out.

The load capacity of the steel arch support was calculated in line with the standard applicable to the design of steel structures in civil engineering. The load capacity was calculated for the other ultimate bending moments that those resulting from the standard. The reason is that years of experience have taught us a lesson: if the standards are complied with rigorously, the design is not economical, this being particularly the case of mines. That is why the software application can use any of the both methods. The steel sections were classified as the 1<sup>st</sup> class sections. They were not, however, classified pursuant to calculations used in the standards because those procedures do not correspond exactly to such complex sections. Classification is based on bending tests [8] and model calculations [7]. Information from bending tests of March 2016 [10] was used too.

A closed steel support behaves as a yielding or non-yielding material. Such behaviour is, among others, depends on the geometry, weight degree (the section), steel grade, quality, workmanship and tightening of fasteners as well as by the way how the support is loaded. In this approach it is assumed that the sliding resistance, this means the resistance against slippage, of the connected support segments is known. The values of resistance should be obtained in slippage tests performed in straight or bent rods.

The loading depends on the driving and supporting technologies and mainly on conditions where the mining work is located. The calculation took also into account the interaction between the closed steel supported and neighbouring rock. A Winkler's model was used.

The second part of the Operation Manual provides operation instructions with respect of the software which was developed using Visual Basic in Microsoft Excel. Attention is paid to the entry of input data, retrieval and use of results and examples of some solutions. The end of the second part explains the use of the archived input data.

## **2. The static solution of a closed steel support using the 1<sup>st</sup> order theory**

A closed steel support is made from various weight sections. Typically, the support consists of at least three circular or straight segments that are connected with each other with screw bolt joints or U-bolt joints. Structures of the steel arch supports which are regularly supplied are described in data sheets provided by the manufactures. It is, however, often necessary to use untypical structures in certain situations. In most cases it is necessary to determine the load capacity – this needs to be made often flexibly for various loads. The same applies to the closed supports.

Each segment of a closed support is defined by its length, radius of curvature and cross-section which is typically the same for all arch supports. Sometimes, there are two radii of curvature in the support segment. A part of the support segment might be sometimes even straight. A straight segment is defined by its length and the radius of

curvature is theoretically infinite. In practice, the radius of curvature is, however, so big so that an error resulting from curvature might be absolutely negligible in the calculation.

For purposes of calculation, following assumptions are made in this chapter: the closed steel support is continuously loaded with the vertical and horizontal loads. The horizontal-to-vertical load ratio,  $\varepsilon$ , may vary considerably and differs on conditions. It can be selected as a parameter ranging from  $\varepsilon = 0$  to  $\varepsilon = \infty$ , where  $\varepsilon = 0$  means that the support is loaded with the continuous vertical load only, while  $\varepsilon = \infty$  means that the support is loaded on both sides with the continuous horizontal load.

The calculation makes it also possible to select a vertical force which is applied onto the closed steel support in a specific place in the level of a curve which is identified by the vertical axis of the closed support.

In addition to the active load, the closed support is also loaded with passive forces. They result from interaction between the support and neighbouring rock mass and fill rock which is located between the solid rock and support. Passive forces result typically from compression of the mining support into base support and may appear along the support perimeter. These are compression forces only.

The calculation consists of three parts:

- Processing of geometry of a closed steel support
- Numerical solution to a static indeterminate structure by a force method for the specified load
- Calculation of internal force components, load capacity and strain components

## 2.1 Geometric analysis of the structure

A good geometric analysis is the basis for the entire solution and is essential for user comfort. Let a circular segment  $j$  have the length  $l_j$  and radius  $r_j$ . Let us assume that the circular segment is covered with the next segment,  $j+1$ , along the  $d_j$  length. In order to describe geometry of the arch structure it is necessary to choose a suitable system of coordinates.

First, a closed steel support needs to be modified in order to describe clearly its geometry. The segment(s) that is/are located next to the vertical axis of the closed support in the bottom surface is reduced virtually by a certain length (ca. 200 mm). See Figure 1. This results in a virtually open support that is limited by **A** on the left side and by **B** on the right side. Let us assume that the both points are located in the same height. Then, it is advisable to choose the beginning of the Cartesian system of coordinates in **A** so that the **x**-axis could pass through **B** and so that the **y**-axis could be vertical (see Fig. 3). If the structure is defined by a number of circular segments, length and radius of curvature of each segment and by the overlap of the neighboring segments, then the line that connects **A** and **B** is not defined directly. It can be, however, clearly defined from the specified values.

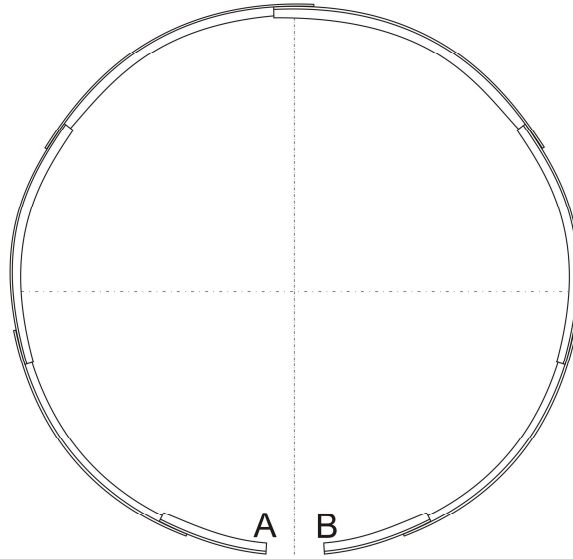


Fig. 1. Virtual open support.

If we choose, for instance, the system of  $x'$ ,  $y'$  coordinates, the beginning of the system will be in A. This means in the left holding point of the support and if the direction of the  $x'$  axis will be parallel to the axis of the first curve,  $x''$  (see Fig. 2), the centre  $S_1$  of the circle of the 1<sup>st</sup> circular segment has the following coordinates:

$$x_1' = r_1 \cdot \cos \frac{\varphi_1}{2} = r_1 \cdot \cos \Phi_1 \quad (2.1)$$

$$y_1' = r_1 \cdot \sin \frac{\varphi_1}{2} = r_1 \cdot \sin \Phi_1 \quad (2.2)$$

where

$$\varphi_1 = \frac{l_1}{r_1} \text{ and } \Phi_1 = \frac{\varphi_1}{2}$$

The coordinates of  $S_2$  which is the centre of the 2<sup>nd</sup> segment (see Fig. 2) in the system of  $x'$ ,  $y'$  coordinates are:

$$x_2' = x_1' + (r_2 - r_1) \cdot \cos \Phi_{21} \quad (2.3)$$

$$y_2' = y_1' - (r_2 - r_1) \cdot \sin \Phi_{21} \quad (2.4)$$

where

$$\Phi_2 = \Phi_1 - \delta_{1,1} \text{ and } \delta_{1,1} = \frac{d_1}{(2 * r_1)}$$

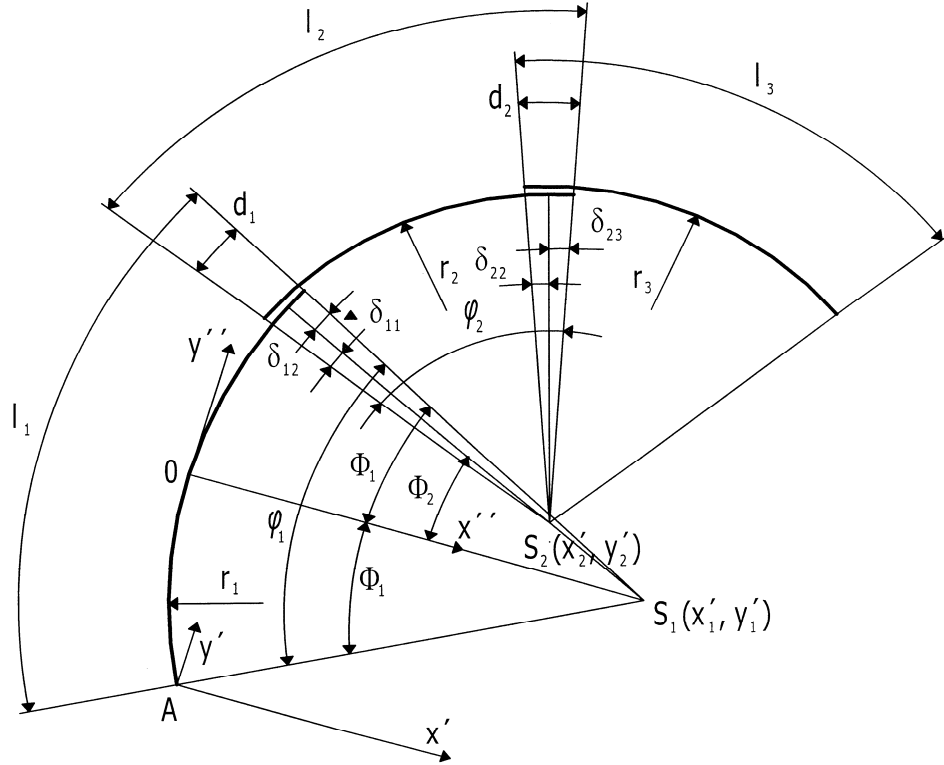


Fig. 2. The coordinates of the 1<sup>st</sup> and 2<sup>nd</sup> segments of the arch in the system of  $x'$ ,  $y'$  coordinates.

In general, the following equations apply to the centre of the circle of the  $j^{\text{th}}$  segment (for  $j > 2$ ) in the system of  $x'$ ,  $y'$  coordinates:

$$x'_j = x'_{j-1} + (r_{j+1} - r_j) \cdot \cos \Phi_j \quad (2.5)$$

$$y'_j = y'_{j-1} - (r_{j+1} - r_j) \cdot \sin \Phi_j \quad (2.6)$$

where

$$\Phi_j = \Phi_{j-1} + \varphi_{j-1} - (\delta_{j-2,j-1} + \delta_{j-1,j-1})$$

$$\delta_{j-2,j-1} = \frac{d_{j-2}}{2 \cdot r_{j-1}} \quad \delta_{j-1,j-1} = \frac{d_{j-1}}{2 \cdot r_{j-1}} \quad \varphi_{j-1} = \frac{l_{j-1}}{r_{j-1}}$$

The coordinates  $x'_B$  and  $y'_B$  for **B**, i.e. the second hinged holders of the support (Fig. 3) are:

$$x'_B = x'_m - r_m \cdot \sin \alpha'_B \quad (2.7)$$

$$y'_B = y'_m - r_m \cdot \cos \alpha'_B \quad (2.8)$$

where  $\alpha'_B = \Phi_n + \varphi_n - \delta_{n-1,n} + \pi/2$  and  $m$  is the total number of circular segments of the support (in Fig 3  $m = 5$ ).

The systems of  $x', y'$  and  $x, y$  are slewed towards each other by  $\omega$ , that angle can be calculated as follows:

$$\omega = \text{arctg} \frac{y'_B}{x'_B} \quad (2.9)$$

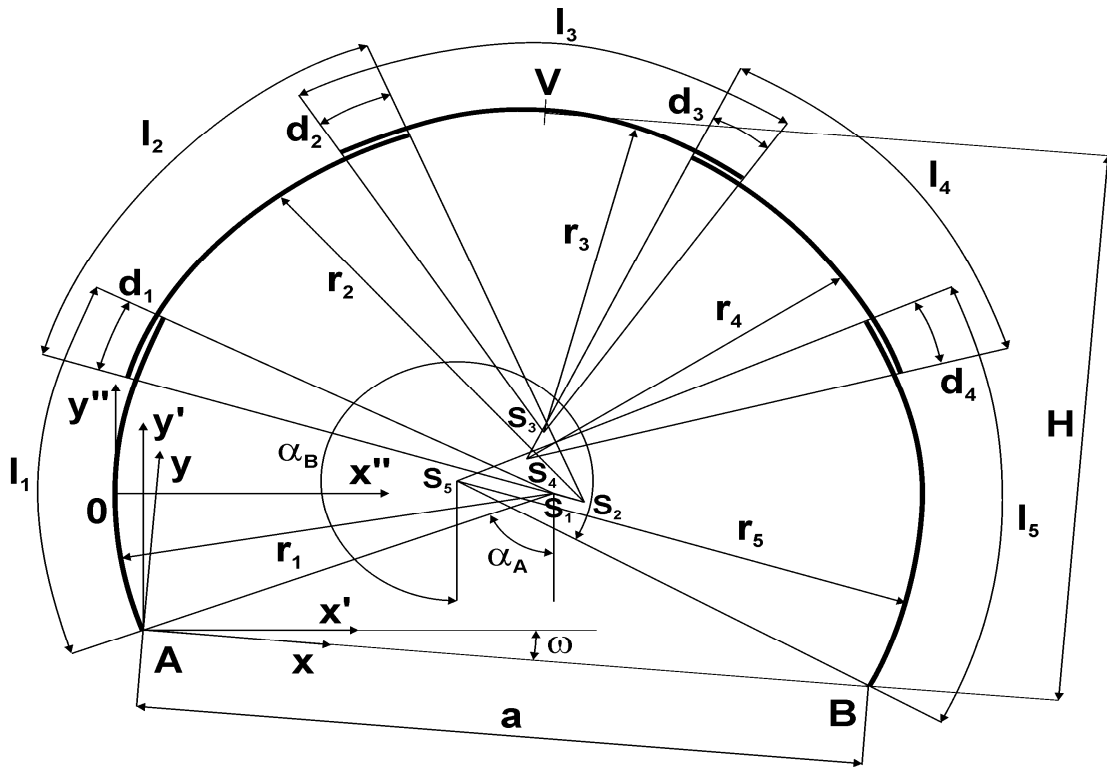


Fig. 3. The geometrical scheme of the system of circular arcs.

The coordinates of the centers of the circles  $S_j$  for each circular segment of the virtual arch support can be transformed in the system of  $x, y$  coordinates using the known equations

$$x_j = x'_j \cdot \cos \omega + y'_j \cdot \sin \omega \quad (2.10)$$

$$y_j = y'_j \cdot \cos \omega - x'_j \cdot \sin \omega \quad (2.11)$$

If the coordinates of the circle centres are known for each segment of the virtual arch support, it is possible to describe accurately the geometry of the structure which is essential for the static solution.

The input values for the geometry of an arch which consists of several circular segments are, as mentioned above, the number of segments  $m$  and, for each  $j$  segment, the length  $l_j$  of such segment, the radius of curvature  $r_j$  and the length of overlap of the neighbouring segments  $d_j$ . Because the points A and B are located in the same height, geometry of the virtual arch is clearly defined.



Once the coordinates of A and B in the virtual arch are known, it is possible to complete description of geometry of the closed support. A part of the closed circular support that is taken between A and B has the specified radius of curvature and the specified length. The vertical axis of the closed support crosses the arch between A and B in D (see Fig. 4). The coordinates of D are as follows:

$$x_D = a / 2 \quad y_D = R_{AB} \cdot (\cos \alpha_{AD} - 1) \quad (2.12)$$

where 
$$\alpha_{AD} = \arcsin \frac{x_D}{R_{AB}}$$

The coordinates of 0 and N result from 2.13:

$$\begin{aligned} x_0 &= x_D - R_{AB} \cdot \sin \frac{\alpha_{AD}}{2} & y_0 &= y_D + R_{AB} \cdot (1 - \cos \frac{\alpha_{AD}}{2}) \\ x_N &= x_D + R_{AB} \cdot \sin \frac{\alpha_{AD}}{2} & y_N &= y_D + R_{AB} \cdot (1 - \cos \frac{\alpha_{AD}}{2}) \end{aligned} \quad (2.13)$$

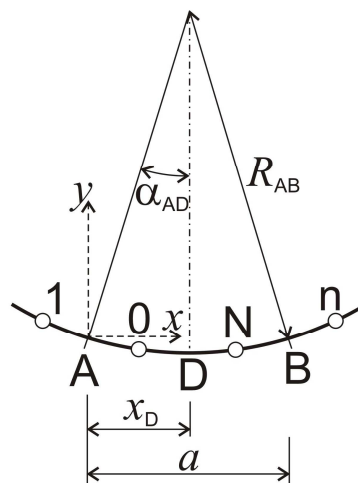


Fig. 4. Closing of the virtually open support.

## 2.2 Static solution to a closed steel support

Several approaches to the static solutions exist now for the closed steel supports. In this case, a standard force method was chosen. And a user does not need to purchase an expensive software. It is enough to have Excel which is installed now in every PC.

The closed support is three times internally statically indeterminate and n times externally statically indeterminate. The internal static indeterminacy results from internal forces as well as from passive forces that are induced by the support that deforms if subject to active loads.



one knows the coordinates of each circular segment in the arch and the segment length  $\Delta s_i$ . The same applies to the central line's tangent in that point.

The statically indeterminate forces can be derived from deformation conditions. These conditions are specified for D (three conditions) and for all centers of  $n$  segments. This means, there are  $n+3$  deformation conditions (2.14) where the statically indeterminate forces  $X_{-3}, X_{-2}, X_{-1}, X_1, \dots, X_i, \dots, X_n$  occur.

$$\begin{aligned}
\delta_{3,-3} \cdot X_{-3} + \delta_{3,-2} \cdot X_{-2} + \delta_{3,-1} \cdot X_{-1} + \delta_{3,1} \cdot X_1 + \delta_{3,2} \cdot X_2 + \dots + \delta_{3,i} \cdot X_i + \dots + \delta_{3,n} \cdot X_n + \delta_{n-3,0} &= 0 \\
\delta_{2,-3} \cdot X_{-3} + \delta_{2,-2} \cdot X_{-2} + \delta_{2,-1} \cdot X_{-1} + \delta_{2,1} \cdot X_1 + \delta_{2,2} \cdot X_2 + \dots + \delta_{2,i} \cdot X_i + \dots + \delta_{2,n} \cdot X_n + \delta_{n-2,0} &= 0 \\
\delta_{1,-3} \cdot X_{-3} + \delta_{1,-2} \cdot X_{-2} + \delta_{1,-1} \cdot X_{-1} + \delta_{1,1} \cdot X_1 + \delta_{1,2} \cdot X_2 + \dots + \delta_{1,i} \cdot X_i + \dots + \delta_{1,n} \cdot X_n + \delta_{n-1,0} &= 0 \\
\delta_{1,-3} \cdot X_{-3} + \delta_{1,-2} \cdot X_{-2} + \delta_{1,-1} \cdot X_{-1} + \delta_{1,1} \cdot X_1 + \delta_{1,2} \cdot X_2 + \dots + \delta_{1,i} \cdot X_i + \dots + \delta_{1,n} \cdot X_n + \delta_{1,0} &= d_1 \\
\delta_{2,-3} \cdot X_{-3} + \delta_{2,-2} \cdot X_{-2} + \delta_{2,-1} \cdot X_{-1} + \delta_{2,1} \cdot X_1 + \delta_{2,2} \cdot X_2 + \dots + \delta_{2,i} \cdot X_i + \dots + \delta_{2,n} \cdot X_n + \delta_{2,0} &= d_2 \quad (2.14) \\
\delta_{3,-3} \cdot X_{-3} + \delta_{3,-2} \cdot X_{-2} + \delta_{3,3} \cdot X_{-1} + \delta_{3,1} \cdot X_1 + \delta_{3,2} \cdot X_2 + \dots + \delta_{3,i} \cdot X_i + \dots + \delta_{3,n} \cdot X_n + \delta_{3,0} &= d_3 \\
&\vdots \\
\delta_{j,-3} \cdot X_{-3} + \delta_{j,-2} \cdot X_{-2} + \delta_{j,-1} \cdot X_{-1} + \delta_{j,1} \cdot X_1 + \delta_{j,2} \cdot X_2 + \dots + \delta_{j,i} \cdot X_i + \dots + \delta_{j,n} \cdot X_n + \delta_{j,0} &= d_j \\
\delta_{n,-3} \cdot X_{-3} + \delta_{n,-2} \cdot X_{-2} + \delta_{n,-1} \cdot X_{-1} + \delta_{n,1} \cdot X_1 + \delta_{n,2} \cdot X_2 + \dots + \delta_{n,i} \cdot X_i + \dots + \delta_{n,n} \cdot X_n + \delta_{n,0} &= d_n
\end{aligned}$$

The equations (2.14) can be also described as a matrix:

$$|\delta| \cdot |\mathbf{X}| + |\delta_0| = |\mathbf{d}| \quad (2.15)$$

$|\delta|$  is a square matrix where the elements can be derived using the Maxwell-Mohr method.

If influence of normal forces and shifting force on strain are not taken into account, then

$$\delta_{ji} = \int_0^l \frac{M_j \cdot M_i}{E \cdot J} \cdot dx \quad (2.16)$$

$M_j$  and  $M_i$  are the moments in  $i \in \langle 1, \dots, i, \dots, n \rangle$  that result from the unit forces  $X_{-1}=1$  applied in  $j \in \langle 1, \dots, i, \dots, n \rangle$  and  $X_{-3}=1, X_{-2}=1$  a  $X_{-1}=1$  in D.  $E$  is the modulus of elasticity of steel and  $J$  is the moment of inertia.

Once the arch is divided into a sufficient number of segments,  $n$ , integration (2.16) can be replaced by summing (2.17):

$$\delta_{ji} = \sum_{p=1}^n \frac{M_{jp} \cdot M_{pi}}{E \cdot J_j} \cdot \Delta s_j \quad (2.17)$$

$|\delta|$  can be calculated as follows:

$$|\delta| = |\mathbf{B}|^T \cdot |\mathbf{M}| \quad (2.18)$$

The rectangular matrix  $[\mathbf{M}]$  has  $n+3$  rows and  $n$  columns. In the  $j^{\text{th}}$  row of the matrix the vector is as follows.

$$|\mathbf{M}_j| = |M_{-3,j} M_{-2,j} M_{-1,j} M_{1,j} M_{2,j} \dots M_{i,j} \dots M_{n,j}| \quad (2.19)$$

In case of the vector (2.19), such indices  $i$  a  $j$  are chosen for the moment  $M_i$ , where  $i$  is the place when the unit force is applied and  $j$  is the place where the unit force becomes apparent.

The rectangular matrix  $[\mathbf{B}]$  has also  $n+3$  rows and  $n$  columns. In the  $j^{\text{th}}$  row of the matrix the vector is as follows.

$$|\mathbf{B}_j| = |B_{-3,j} B_{-2,j} B_{-1,j} B_{1,j} B_{2,j} \dots B_{ij} \dots B_{n,j}| \quad (2.20)$$

$$B_{ij} = \frac{M_{ij}}{E \cdot J_j} \cdot \Delta s_j \quad (2.21)$$

$[\delta]$  is a square matrix with  $n+3$  rows and columns. It corresponds to the degree of static indeterminacy and to the number of unknown static variables that are described in (2.15) by means of the column matrix  $[\mathbf{X}]$ .

In (2.15) there is also the column matrix  $[\delta_0]$  that describes the shift of centres of all segments in the basic static determinate system.

This shift  $\delta_{0,j}$  in  $j$  is generally given by the shift  $\delta_{01,j}$  that results from the loading of the basic statically determinate systems and by the shift  $\delta_{02,j}$  that results from the shift of supports in  $O$  and  $N$  in the basic static determine system. One obtains:

$$|\delta_0| = |\delta_{01}| + |\delta_{02}| + |\delta_{03}| \quad (2.22)$$

The equation for the column matrix  $[\delta_{01}]$  is as follows:

$$|\delta_{01}| = |\mathbf{B}|^T \cdot |\mathbf{M}_0| \quad (2.23)$$

where  $|\mathbf{B}|^T$  results from the equation above and  $|\mathbf{M}_0|$  is the column matrix. It is caused by the active continuous load. Such load can be the continuous unit load  $q_1=1$  or the continuous horizontal load  $q_{1h}=1$  or the load cause by the axial force  $F$ , see Fig. 5.

The shift that is described by the column matrix  $[\delta_{02}]$  is the consequence of a drop of supports in the direction of the axis  $y$  in the basic statically determinate system. And the shift that is described by the column matrix  $[\delta_{03}]$  results from shift of the support  $O$  in the direction of the axis  $x$ .

The unit forces  $X_{-1}=1$  induce in  $O$  in ZS the vertical reaction  $R_{0,j}$  and the horizontal reaction  $H_{0,i}$ . In  $N$  they induce the vertical reaction  $R_{N,j}$  and the horizontal reaction  $H_{N,i}=0$ .

The force  $X_{-j}=1$  and moment  $X_{-3}=1$  induce zero reactions. In line with the principle of virtual work, the strain of centres of each beam segment can be described as follows:

$$\delta_{02,j} = -(R_{0,i} \cdot w_0 + R_{N,i} \cdot w_N) \quad (2.24)$$

$$\delta_{03,j} = -H_{0,i} \cdot u_0 \quad (2.24a)$$

The resulting reaction  $H_0$  in the direction of the axis  $x$  is zero in the point 0. The same is true for the shift  $u_0$  in this point, if the closed support is loaded symmetrically. It is non-zero, when the load is not symmetrical. Its value results from the condition of the zero resulting horizontal reaction  $H_0$ . Calculation of the displacement  $u_0$  is performed iteratively.

The shifts  $w_0$  and  $w_N$  are the result of the loading of the statically indeterminate system. These are vertical shifts in  $\mathbf{0}$  and  $\mathbf{N}$ . Then:

$$R_0 = -k_0 \cdot w_0 \quad \text{and} \quad R_N = -k_N \cdot w_N \quad (2.25)$$

It is assumed that the positive direction of shift and the reaction correspond to the direction of the  $y$ -axis. Direction of the reactions in  $\mathbf{0}$ ,  $\mathbf{N}$  and, in general, the direction of passive forces  $\mathbf{X}$  in  $\mathbf{j}$  is opposite to that of the shift. That is why a negative sign is in (25). The shift of supports can be derived from (2.25):

$$w_0 = -\frac{R_0}{k_0} \quad w_N = -\frac{R_N}{k_N} \quad (2.26)$$

$R_0$  and  $R_N$  are total reactions in  $\mathbf{0}$  and  $\mathbf{N}$  in the static indeterminate structure. Magnitude of the reactions is defined as follows:

$$\begin{aligned} R_0 &= R_{01} + R_{0,-2} \cdot X_{-2} + R_{0,1} \cdot X_1 + R_{0,2} \cdot X_2 + \dots + R_{0,i} \cdot X_i + \dots + R_{0,n} \cdot X_n \\ R_N &= R_{N,1} + R_{N,-2} \cdot X_{-2} + R_{N,1} \cdot X_1 + R_{N,2} \cdot X_2 + \dots + R_{N,i} \cdot X_i + \dots + R_{N,n} \cdot X_n \end{aligned} \quad (2.27)$$

In (2.27)  $R_{0,i}$  and  $R_{N,i}$  are the reactions induced by the unit force  $\mathbf{X}_i$  applied in  $\mathbf{i}$ . After substitution into (2.24):

$$\delta_{02,j} = R_{0,j} \cdot \frac{R_0}{k_0} + R_{N,j} \cdot \frac{R_N}{k_N} = \frac{R_{0,j}}{k_0} \cdot \left( R_{01} + \sum_{i=3}^n (R_{0,i} \cdot X_i) \right) + \frac{R_{N,j}}{k_N} \cdot \left( R_{N1} + \sum_{i=3}^n (R_{N,i} \cdot X_i) \right) \quad (2.28)$$

The shift  $\delta_{02,j}$  can be divided as follows:

$$\delta_{02,j} = \delta_{02,j}^* + \Delta_{02,j}^{**} \quad (2.29)$$

In (2.29) the following holds for  $k_0 = k_N$ :

$$\delta_{02,j}^* = \frac{R_{0,j}}{k_0} \cdot (R_{01}) + \frac{R_{N,j}}{k_N} \cdot (R_{N1}) = \frac{R_{0,j} \cdot R_{01} + R_{N,j} \cdot R_{N1}}{k_0} \quad (2.30)$$

It is obvious that  $|\delta_{02}^*|$  is a column matrix. The same holds for  $|\Delta_{02}^{**}|$ . In the  $j^{\text{th}}$  row it is calculated pursuant to (2.31).

$$\Delta_{02,j}^{**} = \frac{R_{0,j}}{k_0} \cdot \left( \sum_{i=-3}^n (R_{0,i} \cdot X_i) \right) + \frac{R_{N,j}}{k_N} \cdot \left( \sum_{i=-3}^n (R_{N,i} \cdot X_i) \right) \quad (2.31)$$

The matrix  $|\Delta_{02}^*|$  is a product of a square matrix  $|\delta_{02}^{**}|$  and a column matrix of static unknown variables  $|X|$ . In the  $j^{\text{th}}$  row in the matrix  $|\delta_{02}^{**}|$  there are the following elements:

$$\delta_{02,j}^{**} = \frac{(R_{0,j} \cdot R_{0,-3} + R_{N,j} \cdot R_{N,-3})}{k_0}, \dots, \frac{(R_{0,j} \cdot R_{0,i} + R_{N,j} \cdot R_{N,i})}{k_0}, \dots, \frac{(R_{0,j} \cdot R_{0,n} + R_{N,j} \cdot R_{N,n})}{k_0} \quad (2.32)$$

It is assumed that  $k_0 = k_N$ .

The column matrix  $|\Delta_{02}^*|$  can be calculated using (2.33):

$$|\Delta_{02}^{**}| = |\delta_{02}^{**}| \cdot |X| \quad (2.33)$$

$|\delta_{02}^{**}|$  is a  $(n+3) \cdot (n+3)$  square matrix.

The yielding of the  $k_0$ ,  $k_N$  and  $k_j$  segments is general given by the following product

$$k_j = C_j \cdot b \cdot \Delta s_j \quad (2.34)$$

In (2.34)  $C_j$  is the yielding coefficient – specified typically in  $[\text{kN}/\text{m}^3]$ ,  $b$  is the width of contact between the steel support and solid rock/fill rock cushion and  $\Delta s_j$  is the length of the  $j^{\text{th}}$  segment of the support.

$|d|$  from equation (2.15) is a column matrix that describes the shift in all centres of the segments. This can be described using (2.35)

$$|d| = |D_d| \cdot |X| \quad (2.35)$$

If the Winkler's model is used,  $|D_d|$  is a square matrix with non-zero members on the main diagonal only. In the 1<sup>st</sup> row on the diagonal, there are reverse values of  $k_j$ , this means  $1/k_j$ .  $k_j$  in the  $j$  row is specified in (2.34). This does not hold for the first three rows where even the diagonal elements in the  $|D_d|$  matrix are zero.

The following holds for the shift  $w_j$

$$w_j = - \frac{X_j}{k_j} \quad (2.36)$$

After substitution, (2.15) can be modified as follows:

$$\begin{aligned} |\delta| \cdot |X| + |\delta_{01}| + |\delta_{02}| + |\delta_{03}| &= |d| \\ |\delta| \cdot |X| + |\delta_{01}| + |\delta_{02}^*| + |\Delta_{02}^{**}| + |\delta_{03}| &= |D_d| \cdot |X| \\ |\delta| \cdot |X| + |\delta_{01}| + |\delta_{02}^*| + |\delta_{03}| + |\delta_{02}^{**}| |X| &= |D_d| \cdot |X| \end{aligned} \quad (2.37)$$

The static indeterminate quantities are calculated using the following system of linear equations:

$$\begin{aligned} \left\| \delta \right\| + \left\| \delta_{02}^{**} \right\| - \left\| D_d \right\| \cdot \left\| X \right\| &= -\left\| \delta_{01} \right\| - \left\| \delta_{02}^* \right\| - \left\| \delta_{03} \right\| \\ \left\| X \right\| &= \left\| \delta \right\| + \left\| \delta_{02}^{**} \right\| - \left\| D_d \right\|^{-1} \cdot \left( -\left\| \delta_{01} \right\| - \left\| \delta_{02}^* \right\| - \left\| \delta_{03} \right\| \right) \end{aligned} \quad (2.38)$$

The statically indeterminate quantities cannot be calculated explicitly and unambiguously from the system of linear equations (2.37). The forces  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_j, \dots, \mathbf{X}_{n-1}$  and  $\mathbf{X}_n$  are only compression forces. This means, they are positive. Tensile forces may appear in the solution too. They are however non-realistic.

Tensile forces cannot exist at the contact between the solid rock and support. In that case, the calculation needs to be repeated with another values for  $\mathbf{k}_j$ .  $\mathbf{k}_j$  cannot be zero. There would be indeterminate terms in  $\left\| D_d \right\|$ . In places where tensile forces appear,  $\mathbf{k}_j$  should be sufficiently low, for instance  $k_j = 0.0000001$ . The corresponding force is then the tensile force, i.e.  $\mathbf{X}_j < 0$ . Its value is, however, negligible in that case. It is also possible to test if a positive force occurs during the repeated calculations. Then, it would be possible to reach a more precise value for  $\mathbf{k}_j$ .

When calculating the passive forces in the Winkler's model's model, it is necessary to calculate  $\mathbf{k}_j$  using (2.33). It is advisable to make formal changes in the equation in order to simplify the entry of input data for numerical calculations (2.39):

$$k_j = C_j \cdot b \cdot \Delta s_j = D_{c,j} \cdot \Delta s_j \quad \text{where} \quad D_{c,j} = C_j \cdot b \quad (2.39)$$

In order to determine  $\mathbf{k}_j$ , one should know the yielding coefficient  $\mathbf{C}_j$ , the length of the support segment  $\Delta s_j$  and the width of contact between the steel support  $\mathbf{b}$  and solid rock / fill rock cushion. The length  $\Delta s_j$  can be easily calculated. It is however more difficult to determine  $\mathbf{b}$  and, particularly,  $\mathbf{C}_j$ . The best solution would be to measure  $\mathbf{k}_j$  directly on site or to determine it indirectly using the strain and force parameters in the mining working. Such measurements are not, however, available. So, another approach needs to be used in order to determine  $\mathbf{C}_j$  and  $\mathbf{b}$ .

Using the Galerkin's approach [11], the yield coefficient  $\mathbf{C}$  is as follows:

$$C = \frac{E}{R(1 + \mu)} \quad (2.40)$$

where  $\mathbf{E}$  is the modulus of elasticity of the rock or material at the contact between the support and solid rock,  $\mathbf{R}$  is an idealised radius of curvature of the support between the support and solid rock and  $\mu$  is the coefficient of transverse deformation. The yield coefficient that is determined using the approach above might be rather inaccurate. For instance, the modulus of elasticity  $\mathbf{E}$  will not, definitely, include rock ruptures that always

occur behind the under-workings and mining workings and driven by means of the heading machines. (2.40) does not include imperfections and irregularities of the contact between the solid rock and support.  $b$  depends on the section. The width of flanges at the contact between the support and neighbouring environment depends on the section. The table below lists  $C$  and  $D_c$  for different  $b$ . This corresponds to the width of the section flanges K24 ( $b=0,0367$  m) TH29 ( $b=0,034$  m) and TH34 ( $b=0,0425$  m). The sections can be, however, placed on plates that almost do not deform. Then  $D_c$  will be considerably higher. Table 1 lists the values calculated for  $b=0.1$  m and  $b=0.15$  m.

The idealized radius of curvature  $R$  is the radius of curvature of the segments in the mining support placed in the bottom for the supports MP4 ( $R=2600$  mm), MP1 ( $R=3880$  mm) and RP4 ( $R=7500$  mm). The values in the table are approximate only. Extensive experiments would be needed in order to reach precise values.

Hornina		E [MPa]	$\mu$	C [kNm <sup>-3</sup> ]			D <sub>c</sub> [kNm <sup>-2</sup> ] pro b=0,0367 m		
				R=2,6 m	R=3,88 m	R=7,5 m	R=2,6 m	R=3,88 m	R=7,5 m
Krovnina E,F,G,H,I	Hanging wal	140	0,35	39886	26728	13827	1464	981	507
Krovnina A,B,C,D	Hanging wal	430	0,2	137821	92354	47778	5058	3389	1753
Premog (0-0,5)	Coal	480	0,25	147692	98969	51200	5420	3632	1879
Premog (0,5-1)	Coal	480	0,3	142012	95163	49231	5212	3492	1807
Jal.premog III	Coal mixed with clay	375	0,35	106838	71592	37037	3921	2627	1359
Talnina I	Footwal	2917	0,3	863018	578311	299179	31673	21224	10980
Hornina		E [MPa]	$\mu$	D <sub>c</sub> [kNm-2] pro b=0,034 m			D <sub>c</sub> [kNm-2] pro b=0,0425 m		
				R=2,6 m	R=3,88 m	R=7,5 m	R=2,6 m	R=3,88 m	R=7,5 m
Krovnina E,F,G,H,I	Hanging wal	140	0,35	1356	909	470	1695	1136	588
Krovnina A,B,C,D	Hanging wal	430	0,2	4686	3140	1624	5857	3925	2031
Premog (0-0,5)	Coal	480	0,25	5022	3365	1741	6277	4206	2176
Premog (0,5-1)	Coal	480	0,3	4828	3236	1674	6036	4044	2092
Jal.premog III	Coal mixed with clay	375	0,35	3632	2434	1259	4541	3043	1574
Talnina I	Footwal	2917	0,3	29343	19663	10172	36678	24578	12715
Hornina		E [MPa]	$\mu$	D <sub>c</sub> [kNm-2] pro b=0,1 m			D <sub>c</sub> [kNm-2] pro b=0,15 m		
				R=2,6 m	R=3,88 m	R=7,5 m	R=2,6 m	R=3,88 m	R=7,5 m
Krovnina E,F,G,H,I	Hanging wal	140	0,35	3989	2673	1383	5983	4009	2074
Krovnina A,B,C,D	Hanging wal	430	0,2	13782	9235	4778	20673	13853	7167
Premog (0-0,5)	Coal	480	0,25	14769	9897	5120	22154	14845	7680
Premog (0,5-1)	Coal	480	0,3	14201	9516	4923	21302	14274	7385
Jal.premog III	Coal mixed with clay	375	0,35	10684	7159	3704	16026	10739	5556
Talnina I	Footwal	2917	0,3	86302	57831	29918	129453	86747	44877

Table 1

## 2.3 Calculation of the components for the internal force components, capacity and strain

The components of the internal force and strain caused by a specific load are calculated for the centre of each segment. Standard static methods should be used for numerical calculation of components of the internal forces in  $i$  ( $M_{1i}$ ,  $N_{1i}$ ,  $V_{1i}$ ) caused only by the vertical load  $q_1 = 1$ , only by the horizontal load  $q_{h1} = 1$  ( $M_{2i}$ ,  $N_{2i}$ ,  $V_{2i}$ ) or only by  $F$  ( $M_{Fi}$ ,  $N_{Fi}$ ,  $V_{Fi}$ ) in a statically indeterminate closed support. For  $\varepsilon$ , the components of the vertical forces are obtained by superposition:



$$\begin{aligned}
M_{ci} &= M_{Fi} + \kappa \cdot (M_{1i} + \varepsilon \cdot M_{2i}) = M_{Fi} + \kappa \cdot M_i \\
N_{ci} &= N_{Fi} + \kappa \cdot (N_{1i} + \varepsilon \cdot N_{2i}) = N_{Fi} + \kappa \cdot N_i \\
V_{ci} &= V_{Fi} + \kappa \cdot (V_{1i} + \varepsilon \cdot V_{2i}) = V_{Fi} + \kappa \cdot V_i
\end{aligned}
\tag{2.41}$$

In (2.41),  $\kappa$  corresponds to the load capacity of the support. See below.

This solution is based on basic assumptions applicable to an elasticity approach, i.e.

- Geometric linearity – only the designed linearity of the structure is considered. It is assumed with respect to the strain that the strain is so little that it does not influence the size of internal forces and moments in a closed support.
- Physical linearity – linear dependence is assumed between the stress and strain.

A standard elastic approach can be used for calculation of the load capacity of the support. In that case, the following condition needs to be fulfilled for each point in the steel support:

$$\frac{|N_{ci}| \cdot \gamma_{M0}}{A \cdot R_e} + \frac{|M_{ci}| \cdot \gamma_{M0}}{W_x \cdot R_e} \leq 1
\tag{2.42}$$

where

- $N_{ci}$  is the normal force in the cross section  $i$  that is caused in the vertical direction by the force  $F$  and by the continuous load  $q = \kappa \cdot q_l$  and in the horizontal direction by the continuous load  $q_h = \varepsilon \cdot q = \varepsilon \cdot \kappa \cdot q_l$ ,
- $M_{ci}$  is the bending moment in the cross-section  $i$  that is caused in the vertical direction by the force  $F$  and by the continuous load  $q = \kappa \cdot q_l$  and in the horizontal direction by the continuous load  $q_h = \varepsilon \cdot q = \varepsilon \cdot \kappa \cdot q_l$
- $\varepsilon = q_h / q$ , is the horizontal/vertical load ratio
- $\kappa$  is the vertical load coefficient – it is based on the load capacity  $q$  and the specified load  $q_l = 1$ , this means:  $q = \kappa \cdot q_l$
- $A$  is the cross-section area
- $W_x$  is the cross-section modulus
- $R_e$  is the yield limit of steel
- $\gamma_{M0}$  is the partial coefficient of load capacity for the cross section of any grade, this means,  $\gamma_{M0} = 1$  pursuant to Article 1 in ČSN EN 1993-1-1.

With that approach, the load capacity of the support would be too low and the designed support would not be economical. That is why the designing process is based on years of experience and expertise.

The equation (2.42) is, however, used for assessment of the load caused by  $F$  in cases when the support is subject to active zero loads. The assumption is that the support should resist the force even if it were not actively loaded otherwise. Such situation may occur if the solid rock does not load the support. As the same time  $F$  should

not cause permanent deformation in the support. If the condition (2.42) with  $F$  for the zero continuous load  $q$  and  $q_h$  is not fulfilled, it is necessary to consider a decrease in  $F$ . It is also possible to decrease the size, to change the loading point  $x_F$ , to use another support or to create conditions for stable operation of the support. This approach, however, requires individual assessment and validation of the conditions.

The assessment of load capacity of steel arch supports used in underground workings and mines is generally based on ČSN EN 1993-1-1, Eurocode 3: Design of Steel Structures - Part 1-1. General Rules and Rules for Structural Engineering.

When evaluating the load capacity of the supports pursuant to the standard, classification is important for the steel sections TH29, TH34 and K24. According to the results of experiments and evaluation of bending tests in March 2016, the sections can be classified as the class 1 material because plastic hinges can be created there and rotation capacity corresponds to the required plasticity, without reducing the load capacity.

The required reliability for the cross sections loaded with moments, shifting forces or tension/compression can be described pursuant to ČSN EN 1993-1-1 as follows:

$$\left( \frac{N_{Ed}}{N_{pl,Rd}} \right)^2 + \frac{M_{N,Rd}}{M_{pl,Rd}} \leq 1 \quad (2.43)$$

where

- $N_{pl,Rd}$  is the designed plastic capacity of non-reduced cross section in tension
- $M_{pl,Rd}$  is the designed load capacity in bend with respect to any main axis of the cross section
- $N_{Ed}$  is the design axial force, where  $N_{Ed} = N_{ic}$ ,
- $M_{N,Rd}$  is the reduced designed load capacity in bend caused by the axial forces where  $M_{NRd} = M_{ic}$ .

The designed shear force  $V_{Ed}$  pursuant to Article 6.2.10 in the aforementioned standard were below 50% of the design plastic shear capacity  $V_{pl,Rd}$ . This condition is almost always fulfilled, evidence being the parametric calculations. (2.43) can be substituted as follows:

$$\begin{aligned} M_{pl,Rd} &= \frac{W_{pl} \cdot R_e}{\gamma_{M0}} = W_{pl} \cdot R_e \\ N_{pl,Rd} &= \frac{A \cdot R_e}{\gamma_{M0}} = A \cdot R_e \end{aligned} \quad (2.44)$$

In addition, equations (2.44) uses the following parameter  $W_{pl}$  - the plastic modulus of the cross section.

The equation (2.43) can be modified as follows:

$$\left( \frac{\kappa_i \cdot N_i + N_{Fi}}{N_{pl,Rd}} \right)^2 + \frac{\kappa_i \cdot M_i + M_{Fi}}{M_{pl,Rd}} \leq 1 \quad (2.45)$$

and

$$a_i \cdot \kappa_i^2 + b_i \cdot \kappa_i + c_i \leq 0 \quad (2.46)$$

where

$$a_i = N_i^2 \cdot M_{pl,Rd}$$

$$b_i = 2 \cdot N_i \cdot N_{Fi} \cdot M_{pl,Rd} + M_i \cdot N_{pl,Rd}^2$$

$$c_i = N_{Fi}^2 \cdot M_{pl,Rd} + N_{pl,Rd}^2 (M_{Fi} - M_{pl,Rd})$$

$\kappa_i$  can be obtained explicitly and directly from (2.44). This describes the load capacity of the support for transfer of hot compression in the place of support.

The limit state is evaluated in terms of the load capacity for each cross section  $i$  of the steel arch.  $\kappa$  is the minimum value from the set of all  $\kappa_i$  in the cross sections of the steel arch.

Then, the load capacity of the support is:

$$q = \kappa \cdot q_1 = \kappa$$

This load capacity  $q$  holds for non-yielding steel supports loaded pursuant to Fig. 5 with the specified  $\varepsilon$ .

Below are components of internal forces in  $i$  in the steel arch for the  $q$ :

$$M_i = \kappa \cdot (M_{1i} + \varepsilon \cdot M_{2i})$$

$$N_i = \kappa \cdot (N_{1i} + \varepsilon \cdot N_{2i})$$

$$V_i = \kappa \cdot (V_{1i} + \varepsilon \cdot V_{2i})$$

(2.47)

Equation (2.47) describes the total internal forces. The strain in the steel arch support can be determined using the principle of virtual work (the unit force method): a unit force is applied step by step in each point in the support - in  $i$ . The unit force is applied in the specified direction which depends on shifting. Or a unit moment is applied there if angular shifting should be specified. The support is loaded as a statically determinate load where reactions correspond to the specified load capacity of the support.

Shifting  $u_k$  in  $k$  with respect to the x- axis is obtained as follows:

$$u_k = \sum_{i=1}^n \frac{M_{ci} \cdot M_{ki} \cdot \Delta s_i}{E \cdot I_i} \quad (2.48)$$

where  $M_{ki}$  is the bending moment in  $i$  caused by the unit virtual force  $F_{xk} = 1$  applied in the  $x$ -axis towards  $k$ .

Pursuant to (2.48) and (2.49) the influence of the normal force on the shift is neglected in the calculation.

Shifting  $v_k$  in  $k$  towards the  $y$ -axis can be calculated by analogy:

$$v_k = \sum_{i=1}^n \frac{M_{ci} \cdot M_{ki} \cdot \Delta s_i}{E \cdot I_i} \quad (2.49)$$

where:

$M_{ki}$  is the bending moment in  $i$  caused by the unit virtual force  $F_{yk} = 1$  applied in the  $y$ - axis towards  $k$

$N_{ki}$  is the normal force induced in  $i$  caused by the unit virtual force  $F_{yk} = 1$  applied in the  $y$ -axis towards  $k$

The shifts calculated pursuant to (2.46) and (2.47) result from internal forces in a closed static indeterminate support. They do not include shifts caused by subsidence of supports  $O$  and  $N$  pursuant (2.26). The total shift in the direction of x and y axes are as follows:

$$u_{k,c} = u_k + u_{k,p} \quad \text{and} \quad v_{k,c} = v_k + v_{k,p} \quad (2.50)$$

In (2.50),  $u_{k,p}$  and  $v_{k,p}$  are the shifts of  $k$  caused by drop of supports in  $O$  and  $N$ . If the closed steel support is subject to a symmetric load, then

$$u_k = 0 \quad \text{and} \quad v_k = w_0 = w_N \quad (2.51)$$

If the load is not symmetric, the drop of supports in  $O$  and  $N$  is different, this means  $w_0 \neq w_N$ ,  $w_k = u_0$  and the shifts  $u_{k,p}$  and  $v_{k,p}$  are calculated by methods used in structural stress engineering.

The total shift  $\delta_k$  in  $k$  is:

$$\delta_k = \sqrt{(u_k^2 + v_k^2)} \quad (2.52)$$

The shifts calculated by the software are derived for the load capacity of the non-yielding support. If the real load of the support is lower than the load capacity, the real shift will be lower than the calculated one.

## 2.4 Static parameters of TH29, TH34 a K24 sections, ultimate moments and normal forces

The table 2 lists the basic mechanical properties of the steel, 31Mn4, 31Mn4+Qt and H60U (DV-60). Table 3 gives static parameters of TH29, TH34 and K24 sections. The values are taken from manufacturer's data sheets. The plastic cross-section modulus,  $W_{xpl}$ , for TH was obtained by calculation.

Steel grade	$R_e$ [MPa]	$R_m$ [MPa]	$A_5$
31Mn4	min. 350	min.550	18
31Mn4+QT	min. 520	min. 650	18
H60U(DV60)	min. 510	min.690	15

Table 2 ( $R_e$  = yield point,  $R_m$  = ultimate tensile strength,  $A_5$  = ductility)

Marking	$A$ [cm <sup>2</sup> ]	$G$ [kg/m]	$W_x$ [cm <sup>3</sup> ]	$W_y$ [cm <sup>3</sup> ]	$I_x$ [cm <sup>4</sup> ]	$I_y$ [cm <sup>4</sup> ]	$W_{xpl}$ [cm <sup>3</sup> ]
K - 24	30,15	23,67	69,19	65,47	372,37	404,49	97,07
TH- 29	37,00	29,00	94,00	103,00	616,00	775,00	132,38
TH - 34	43,10	33,80	128,00	141,00	892,00	1205,00	177,83

Table 3

The compressive cross-section load capacity and the plastic moment of cross-section capacity which are needed for calculation pursuant to (2.43) are obtained from (2.44):

$$N_{pl,Rd} = \frac{A \cdot R_e}{\gamma_{Mo}} = A \cdot R_e \quad M_{pl,Rd} = \frac{W_{xpl} \cdot R_e}{\gamma_{Mo}} = W_{xpl} \cdot R_e \quad (2.53)$$

Next table 4 summarizes the values for the specified sections and steel grades:

Section	$R_e$ [MPa]	$A$ [cm <sup>2</sup> ]	$W_{xpl}$ [cm <sup>3</sup> ]	$N_{pl,Rd}$ [kN]	$M_{pl,Rd}$ [kNm]
K24/H60U(DV60)	510	30,15	97,07	1537,7	49,51
TH29/31Mn4	350	37,00	132,38	1295,0	46,33
TH29/(31Mn4+QT)	520	37,00	132,38	1924,0	68,84
TH34/31Mn4	350	43,10	177,83	1508,5	62,24
TH34/(31Mn4+QT)	520	43,10	177,83	2241,2	92,47

Table 4

The values are considerably lower than those used in calculations pursuant to [2] where another method was used to calculate the bending moments  $M_{pl}$ . The values were generally based on the standard ČSN 44 4410-1 Testing of Mining Steel Supports - Part 1: Testing of Forming Rods. The standard specifies the minimum loading force for the forming rods K21, K24 a P28 which are made from the 11500 steel grade for bending tests of straight rods. The holding points are located in the distance of 1 m. The load is

applied in the middle of the span in both the inside and outside of the cross-section root. The values taken from the standard for the K21, K24 and P28 sections had to be warranted by the manufacturer and were the basis for the ultimate moments. The compressive strength was obtained from the warranted yield point of steel and cross-section area of the steel section using the equation  $N_{pl} = A \cdot R_e$ . This approach has been used for many years in the Ostrava – Karviná Colliery and it has generally proved itself. This approach considers the fact that the capacity of open sections used as supports in long mining workings is different and depends on direction of load which is not generally considered in standards.

Regarding the TH steel section made from 31Mn4 and 31Mn4+QT, no information is available for the minimum loading force. The values were derived theoretically from the cross-section characteristics of the steel sections, bending tests and properties of the steel grade, from which the TH sections have been manufactured. The same applies to the K24 section made from H60U.

Pursuant to [9] ČSN EN 1993-1-1, Eurocode 3: Design of steel structures – Part 1-1: General Rules and Rules for Structural Engineering, the value of  $M_{pl,Rd}$  for the designed bending load capacity towards the main axis of the cross-section is identical in the both loading directions (see Table 4).

It follows from the bending tests and ČSN 44 4410-1 Testing of Mining Steel Supports that the loading capacity of the open K/TH section is the difference for the inside-the-section load and for the section-root load. The reason is, in particular, various stability of a loaded section.

Section	$M_{pl1}$ [kNm]	$M_{pl2}$ [kNm]	$N_{pl}$ [kN]
K24/H60U	49,51	-62,70	1537,7
TH29/31Mn4	50,85	-64,73	1295,0
TH29/(31Mn4+QT)	75,72	-96,37	1924,0
TH34/31Mn4	68,47	-87,13	1508,5
TH34/(31Mn4+QT)	95,18	-112,45	2241,2

Table 5

The table 5 gives the ultimate plastic moments,  $M_{pl1}$  and  $M_{pl2}$ , for the sections and steel. They are derived from the models and bending tests which were performed in past and in March 2016. The values and the compressive load capacity  $N_{pl}$  are taken from table 4.

The client required that a particular attention should be paid to the static parameters of K24 (made from H60U) after 10%, 20% and 30% corrosion.

The assumption was that the corrosion would appear evenly along the cross-section and will reach generally the specified percentage of the weight and surface in the cross-section.

The table 5 lists the static parameters of K24 with the specified corrosion for that assumption.

Marking	Thickness of corrosion [mm]	A [cm <sup>2</sup> ]	G [kg/m]	J <sub>x</sub> [cm <sup>4</sup> ]	W <sub>x</sub> [cm <sup>3</sup> ]	W <sub>xpl</sub> [cm <sup>3</sup> ]
K24-10% corrosion	0,581	27,17	21,30	339,74	63,60	88,43
K24-20% corrosion	1,165	24,15	18,94	307,43	57,95	79,78
K24-30% corrosion	1,754	21,13	16,57	275,66	52,30	71,16

Table 6

The table 7 lists the ultimate moments of the corroded section K 24 for the specified static parameters.

Section	M <sub>pl,Rd</sub> [kNm]	M <sub>pl1</sub> [kNm]	M <sub>pl2</sub> [kNm]	N <sub>pl</sub> [kN]
K24/H60U-10% corrosion	45,10	45,10	-57,12	1385,7
K24/H60U-20% corrosion	40,69	40,69	-51,53	1231,6
K24/H60U-30% corrosion	36,29	36,29	-45,96	1077,4

Table 7

## 2.5 Load capacity of a yielding support

In case of a yielding steel support the load capacity is achieved if the slippage resistance in places where the segments overlap is lower than the normal force which is caused there by an external load. In this case, the load capacity of a steel arch support is limited, not only by the ultimate state but also by the slippage resistance.

Theoretical calculation of slippage resistance is not only difficult, but also inaccurate. It depends on several factors which are very difficult to model, or even to determine. The slippage resistance depends, in particular, on:

- 1) the structural design of the fasteners
- 2) the number of the fasteners
- 3) the tightening of U-bolts/screws in the fasteners
- 4) the quality of steel of the fasteners
- 5) the structural design of the connected section rods
- 6) quality of steel of the connected section rods
- 7) inaccuracy of the connected section rods
- 8) contact between the connected section rods and joints with braces, inclined props and/or rock

Therefore, it is advisable to use an experimental approach. In the text below the load capacity of the yielding support is based on the specific slippage resistance from 100 kN to 350 kN, the step being 50 kN.

### 3. The software for calculation of the load capacity of closed steel supports

The theoretical background above was used as a basis for development of a software application in Microsoft Excel. The application was developed in Visual Basic. Fig. 6 shows the title page of the software application.

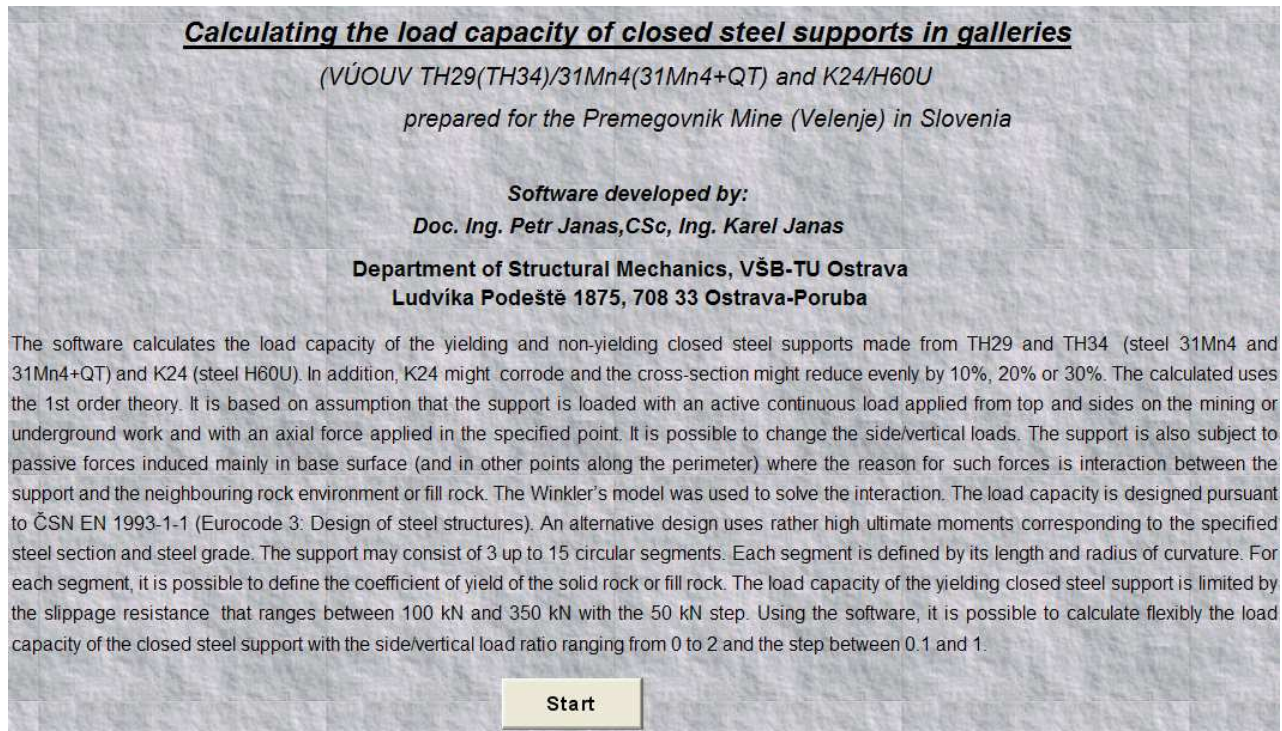


Fig. 6 Title page with description of the software application

The software calculates the load capacity of the non-yielding and yielding closed supports made from steel sections: TH29, TH34 (31Mn4 and 31Mn4+QT) and K24 (H60U). It uses the 1<sup>st</sup> order theory and is based on assumption that the support is placed in two joints and loaded with an active continuous load applied from top and sides on the mining or underground work and one axial force applied from top in the plane of the support. The side/vertical load ration can be freely modified.

The support may consist of 3 up to 15 circular segments with different lengths and arch radius. If a sufficiently high radius of curvature is chosen, it is also possible to simulate straight segments of the support.

#### 3.1 Entry of input data

Once you click on **Start** in the home screen of the software, the input form will appear. See Fig. 7. Before you start entering the input data, click **Next run** (this button will change later into **Run**). Then click **Last entry**. The screen shows the input data for the **Last entry**. These data can be, obviously, changed.



Before the input data are entered for the closed support, it is necessary to create a virtual arch support pursuant to Fig. 1. It is recommended to specify in advance in which direction the segments of the closed support will be numbered. It is recommended to use the clockwise direction where the first segment will start in the lowest point of the closed support. The virtual arch support is created as follows: The arch that is located in the lowest part of the closed support is divided into two separate segments. Each segment will be shortened by ca. 100 mm. The length of each virtual segment will be a half of the original length minus ca. 100 mm. Each virtual segment after division is regarded as a separate segment. If the arches are connected in the lowest closed support, formation of the virtual open support requires to shorten the length of the original and last segments by a half of the overlapping length of the first and last segments and each other segment should be made shorter by another ca. 100 mm.

If the geometry of the virtual arch support is entered incorrectly, there will be no gap between 0 and N. The distance between  $x_N$  and  $x_0$  is negative and more precise data are needed. See Fig. 8.

Following input data are entered in the calculation (See Fig. 7):

- The number of circular arches that form the closed steel support. It is possible to specify 3 up to 15 circular segments.
- The section of each segment. Following sections are available: TH29, TH34 and K24.
- Steel grade. TH29 and TH34 are made from 31Mn4 or 31Mn4+QT. K24 is made from H60U. For this steel grade, it is possible to specify the corrosion, 10%, 20% or 30%, that reduces that cross-section.
- The horizontal-to-vertical load ratio,  $\epsilon$ .
- The value to be specified is generally from 0.0 without any upper limit.
- The length  $L_j$  of each circular segment. If the length of the segment(s) and/or other data are specified incorrectly, the distance between **O** and **N** in the x-axis direction is negative ( $x_N - x_0 < 0$ ). In that case, an error entry message will display. See Fig. 8.
- Radius of curvature  $R_j$  for each segment.

The screenshot shows a software interface for calculating the load capacity of a support. The title bar reads "VUOUV TH29(34)/31Mn4(31Mn4+QT), K24/H60U". The main form contains the following fields and data:

- Calc. No: MP1-K24/H60U
- Number of arches: 8
- Section: K-24
- Steel grade: H60U
- $\epsilon$ : 1

Arch length [mm]	Radius [mm]	Overlapping [mm]	Index Dci [kN/m <sup>2</sup> ]
L1: 800	R1: 3880	O 1-2: 0	Dc1: 3000
L2: 1570	R2: 1500	O 2-3: 480	Dc2: 3000
L3: 2650	R3: 2125	O 3-4: 480	Dc3: 0.0000001
L4: 2650	R4: 2125	O 4-5: 480	Dc4: 0.0000001
L5: 2650	R5: 2125	O 5-6: 480	Dc5: 0.0000001
L6: 2650	R6: 2125	O 6-7: 480	Dc6: 0.0000001
L7: 1570	R7: 1500	O 7-8: 0	Dc7: 3000
L8: 800	R8: 3880	O 8-1: 480	Dc8: 3000

Additional form elements include checkboxes for "EuroCode 3" and "Graph  $\epsilon$  / q", a field for "EuroCode 3  $\epsilon$  step", and input fields for "Joint 1" and "Joint 2" (both set to 0). There is also an "Axial force" section with fields for "F" (0 [kN]) and " $x_P$ " (0 [mm]). Buttons for "Next run", "Last entry", "Results details", "Save entry", and "Archives" are present. The "Date" field is set to 10.10.2016.

Fig. 7. The entry form sheet for calculation of load capacity of a support.

- The length of overlapping for the neighbouring segments. Each segment which has two radii of curvature needs to be entered as two arches (two segments). Each segment has its own length and radius of curvature. The overlapping length for such two segments is zero. If there is an only one segment in the lowest point and that segment is virtually divided in two, the overlapping between the first and last segments is zero. If two segments (the first and last segments) are connected in the lowest point, the overlapping length is not zero (see Fig 7: O 8-1 480 mm).
- For each segment,  $D_{cj}$  is specified from (2.39). Except for the first and last segments (including the virtual ones)  $D_{cj}$  might be zero. Typically,  $D_{cj}$  should be different from zero at least for the segments that are placed on the bottom (see Fig. 7:  $D_{c1}=D_{c2}=D_{c7}=D_{c8}=3000 \text{ kNm}^{-2}$ .  $D_{c3}=D_{c4}=D_{c5}=D_{c6}=0$ )
- The load capacity for TH29 and TH 34 is calculated using the ultimate values from Table 4. If the EuroCode3 checkbox is checked, the load capacity is calculated using the ultimate moments and normal forces from Table 3. The values for K24 are listed in Table 6.
- $F$  [kN] applied in the plane of the support and the  $x_F$  distance [m] (this is the horizontal distance from the axis of the working – see Fig. 5).
- Hinges can be specified in two points of the closed arch support. If 0 is entered, there are no hinges in the support. Numbers of checkboxes where hinges should be placed are not, in general, known upon entering of the input data. This means, at the beginning the calculation is performed without any hinges and the numbers of hinges are specified only after the first calculation when one obtains information about general geometry of the closed steel support and information about centres and coordinates of the segments.
- At the same time, it is possible to calculate the load capacity of the closed support for  $\epsilon$  ranging between 0.0 and 2.00 where the step can be specified between 0.1 and 1.
- This calculation is performed if you check the checkboxes **Graph  $\epsilon$  /q** and  **$\epsilon$  step** in the right pane of the form sheet.
- This calculation is performed with  $F = 0$ .
- For archiving, the calculation # and note can be entered. The date of calculation is generated automatically.
- If you click **Save entry**, the entered data and the note text, if any, will be saved in the archives.
- The input data can be downloaded from the **Archives**, if they were saved there. To do this, click the **Archives**. Before the data are downloaded from the archives, you must click **Next run**. Then, data from the archives can be specified in detail.

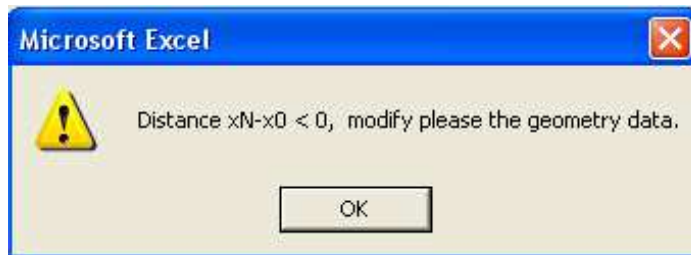


Fig. 8 Information about faulty entry.

### 3.2 Calculation results

Once the calculation is completed, the results are displayed in the form sheet. Fig. 9 shows calculation of the load capacity of the closed steel support MP1 made from the K24 section and H60U steel. The overlapping between the first and eight segments is not zero. This means that the segments located at the lowest level were made shorter virtually. For that calculation,  $\varepsilon=1$  was specified (it describes the vertical load related to the horizontal load).

If compared with the input form, the output form includes more information, namely:

- The height  $H$  is the difference  $H = y_{\max} - y_{\min}$ , this means the distance between the highest and lowest points in the centre line of the closed support.
- The width  $a$  which is the difference  $a = x_{\max} - x_{\min}$ , this means the highest horizontal distance of the centre line of the closed support.
- The table with the load capacities  $q$  for the support in the vertical direction. This corresponds to the total active vertical load  $Q$  and the horizontal load  $q_h$  for  $\varepsilon = 1$  as well as the total active horizontal load  $Q_h$ .
- $T_{\min}$  is the minimum slippage resistance in the place where the support segments overlap and where the steel support behaves as a non-yielding component.  $q$ ,  $q_h$ ,  $Q$  and  $Q_h$  are specified in  $T_{\min}$  column for the non-yielding support. If the slippage resistance is lower than the specified value, the load capacity of the support is limited by the slippage resistance. If the slippage resistance exceeds  $T_{\min}$ , the support does not yield and its load capacity is limiting for the ultimate state pursuant to (2.43) or (2.46). For the slippage resistance being  $T = 100 \text{ kN}$ ,  $150 \text{ kN}$ ,  $200 \text{ kN}$ ,  $250 \text{ kN}$ ,  $300 \text{ kN}$  and  $350 \text{ kN}$ , each column provides, in addition to the slippage resistance, the load capacity and reaction of the support.

The values in the table with the results are valid always for the support, the geometry and section of which are specified in the form sheet (in this case, K24 made from H60U) and for the specified horizontal/vertical load ratio  $\varepsilon$  ( $\varepsilon = 1.0$  in this case) and  $F=0$ . See Fig. 7.

The form sheet informs about the end of calculation and time of calculation **Process:** "End of calculation" [23 sec.].

If you click and confirm **Save entry** (see Fig. 10), the input data incl. the calculation date and notes are saved into **Archives**. To open the **Archives** click Archives.

To start new calculation click **Next run**. Before new or edited entries are made, click **Last entry**. If the entries are specified before you click **Next run** and **Last entry**, the original entries will be used when you click **Next run**. To print the form sheet with the results click **Print results**.

If  $F$  exceeds the ultimate value for the support, the output form sheet shows information displayed in Fig. 10. In that case the calculation is stopped and the load capacity cannot be determined for the continuous load applied from top and sides of the working. In that case click **OK** to edit the entries. It is possible to change  $F$  or  $x_F$  where  $F$  is applied or the geometry of the support. Or another weight level of the profile can be entered.

If the permitted distance  $x_F$  is exceeded, the warning will appear. See Fig. 12. In this case click **OK** and change the data.

VU0UV TH29(34)/31Mn4(31Mn4+QT), K24/H60U

Calc. No: MP1-K24/H60U Number of arches: 8 Section: K-24 Steel grade: H60U  $\epsilon$ : 1

Arch length [mm]	Radius [mm]	Overlapping [mm]	Index Dci [kN/m <sup>2</sup> ]
L1: 800	R1: 3880	O 1-2: 0	Dc1: 3000
L2: 1570	R2: 1500	O 2-3: 480	Dc2: 3000
L3: 2650	R3: 2125	O 3-4: 480	Dc3: 0.0000001
L4: 2650	R4: 2125	O 4-5: 480	Dc4: 0.0000001
L5: 2650	R5: 2125	O 5-6: 480	Dc5: 0.0000001
L6: 2650	R6: 2125	O 6-7: 480	Dc6: 0.0000001
L7: 1570	R7: 1500	O 7-8: 0	Dc7: 3000
L8: 800	R8: 3880	O 8-1: 480	Dc8: 3000

EuroCode 3  
 Graph  $\epsilon$  / q  $\epsilon$  step:

Joint 1:  Joint 2:

Axial force  
 F:  [kN]  
 X<sub>F</sub>:  [mm]

Date: 10.10.2016

Note:

Process: **\*\*\* End of calculation \*\*\* [ 23 sec.]**

**Results**

a:  [m] H:  [m]

	Tmin = 393.14 kN	T = 100 kN	T = 150 kN	T = 200 kN	T = 250 kN	T = 300 kN	T = 350 kN
q [kN/m]	101,755	25,882	38,823	51,764	64,705	77,647	90,588
qh [kN]	101,755	25,882	38,823	51,764	64,705	77,647	90,588
Q	432,435	109,993	164,99	219,986	274,983	329,979	384,976
Qh	403,192	102,555	153,832	205,11	256,387	307,665	358,942

**Explanation:** T ... Sliding resistance  
 Tmin ... Minimum load capacity of unyielding joint

Print results

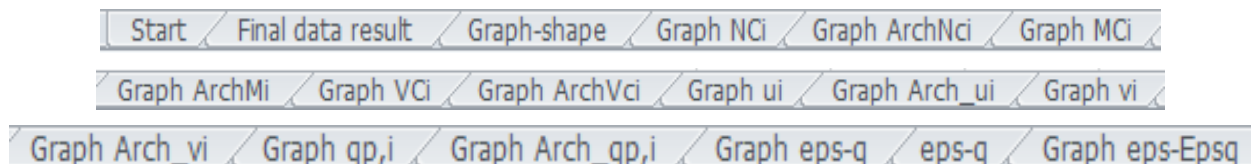


Fig. 9. The output form sheet for calculation of the load capacity of MP1 made from K24/H60U. The results and tabs in the bar.



Fig. 10 Confirmation that the calculation was saved in archives.

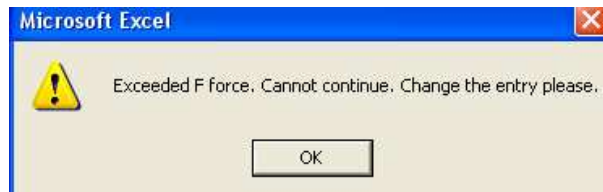


Fig. 11 Information that F was exceeded.

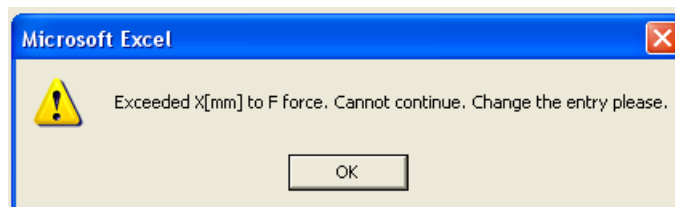


Fig. 12 Information that  $x_F$  was exceeded.

If you click **Result details**, following options are available depending on the tabs in the bar under the output form sheet (see Fig. 9):

- **Start** The home view with description of the software and **Start** button which, once clicked, returns the display to the entry form sheet with the last entries (Fig. 7).
- **Final data result** The table with partial results of calculation. If you click the tab in the bar, the results (except for the support shape) are valid for the specified section, load capacity and horizontal/vertical load ratio  $\varepsilon$ .

The headings of the column (Fig. 13) provide the following values:

- $i$  ... the serial number of the segment,
- $ds_i$  ... the length of  $i$  [mm],
- $l_i$  ... the summary length of all  $i$  segments, it is needed for development of the central line of the closed support into a line. It is calculated from the point D.

$$l_i = \sum_{i=1}^{i-1} ds_i + ds_i / 2$$

- $Ai$  ... the tangent of  $i$ ,
- $X_i, Y_i$  ... the coordinates of the centre of  $i$  [mm],
- $EI_i$  ... the bending rigidity of the support in  $i$  [ $\text{kNm}^2$ ]
- $K_i$  ... the yielding coefficient of  $i$ , it results from (2.34) or (2.39) [ $\text{kNm}^{-1}$ ],
- $M_{ci}$  ... the total bending moment in the  $i^{\text{th}}$  segment once the load capacity of a non-yielding support is achieved [ $\text{kNm}$ ],

i	dS <sub>i</sub> [mm]	l <sub>i</sub> [mm]	A <sub>i</sub>	x <sub>i</sub> [mm]	y <sub>i</sub> [mm]	EI <sub>i</sub> [kNm <sup>2</sup> ]	K <sub>i</sub> [kNm <sup>2</sup> ]	M <sub>Fi</sub> [kNm <sup>-1</sup> ]	V <sub>Fi</sub> [kN]	N <sub>ci</sub> [kN]	M <sub>ci</sub> [kNm <sup>-1</sup> ]	V <sub>ci</sub> [kN]	N <sub>ci</sub> [kN]	q <sub>p,i</sub> [kNm <sup>-1</sup> ]	v <sub>i</sub> [mm]	u <sub>i</sub> [mm]
D	0,00	0,00	0,0205	79,45	-0,81	2871,79	0,00	0,00	0,00	0,00	-11,99	0,00	-295,36	129,88	-35,10	0,00
0	79,45	39,73	0,0102	39,72	-0,61	2871,79	293,98	0,00	0,00	0,00	-11,93	-7,30	-295,41	129,88	-35,10	0,00
1	200,00	179,45	0,0322	-99,98	1,93	2871,79	740,00	0,00	0,00	0,00	-12,62	-13,80	-295,04	129,65	-35,06	0,00
2	200,00	379,45	0,0837	-299,62	13,51	1293,60	740,00	0,00	0,00	0,00	-16,45	-24,54	-293,90	128,37	-34,82	0,01
3	200,00	579,45	0,1353	-498,40	35,36	1293,60	740,00	0,00	0,00	0,00	-22,32	-35,22	-291,45	124,96	-34,07	0,10
4	200,00	779,45	0,1868	-695,79	67,42	1293,60	740,00	0,00	0,00	0,00	-30,04	-45,63	-287,72	118,92	-32,65	0,33
5	181,67	970,29	0,2731	-891,96	109,09	1293,60	672,17	0,00	0,00	0,00	-38,44	-44,50	-284,21	109,28	-30,44	0,82
6	181,67	1151,95	0,3942	-1053,50	168,56	1293,60	672,17	0,00	0,00	0,00	-43,59	-32,66	-278,59	96,52	-27,49	1,85
7	181,67	1333,62	0,5154	-1216,60	248,31	1293,60	672,17	0,00	0,00	0,00	-44,97	-22,25	-268,78	82,98	-23,67	3,71
8	181,67	1515,29	0,6365	-1368,87	347,19	1293,60	672,17	0,00	0,00	0,00	-42,33	-14,42	-255,76	71,61	-19,15	6,65
9	181,67	1696,95	0,7576	-1508,08	463,73	1293,60	672,17	0,00	0,00	0,00	-35,70	-10,58	-240,82	65,61	-14,20	10,80
10	181,67	1878,62	0,8787	-1632,19	596,24	1293,60	672,17	0,00	0,00	0,00	-25,42	-12,27	-225,48	67,71	-9,15	16,18
11	120,00	2029,45	0,9792	-1722,45	717,01	2871,79	444,00	0,00	0,00	0,00	-14,38	-18,76	-213,79	76,75	-5,16	21,52
12	120,00	2149,45	1,0592	-1785,31	819,19	2871,79	444,00	0,00	0,00	0,00	-4,17	-27,92	-205,76	88,45	-2,34	26,10
13	120,00	2269,45	1,1275	-1840,13	925,91	2871,79	0,00	0,00	0,00	0,00	7,20	-36,87	-199,11	0,00	0,12	30,90
14	120,00	2389,45	1,1839	-1888,52	1035,71	2871,79	0,00	0,00	0,00	0,00	18,28	-44,31	-193,25	0,00	2,28	35,81
15	211,25	2555,08	1,2619	-1944,96	1191,37	1293,60	0,00	0,00	0,00	0,00	32,56	-56,44	-187,39	0,00	4,76	42,64
16	211,25	2766,33	1,3613	-1999,09	1395,48	1293,60	0,00	0,00	0,00	0,00	48,41	-74,33	-184,25	0,00	6,85	50,52
17	211,25	2977,58	1,4607	-2032,69	1603,95	1293,60	0,00	0,00	0,00	0,00	60,93	-93,84	-186,50	0,00	7,88	56,92
18	211,25	3188,83	1,5601	-2045,43	1814,73	1293,60	0,00	0,00	0,00	0,00	69,60	-113,69	-194,48	0,00	8,15	61,29
19	211,25	3400,08	1,6595	-2037,19	2025,73	1293,60	0,00	0,00	0,00	0,00	74,19	-132,60	-207,49	0,00	8,07	63,27
20	211,25	3611,33	1,7589	-2008,05	2234,87	1293,60	0,00	0,00	0,00	0,00	74,69	-149,70	-224,19	0,00	8,15	62,70
21	211,25	3822,58	1,8584	-1958,29	2440,09	1293,60	0,00	0,00	0,00	0,00	71,27	-164,25	-244,27	0,00	8,89	59,64
22	211,25	4033,83	1,9578	-1888,42	2639,36	1293,60	0,00	0,00	0,00	0,00	64,24	-175,61	-267,26	0,00	10,75	54,34
23	120,00	4199,45	2,0357	-1820,00	2790,15	2871,79	0,00	0,00	0,00	0,00	56,48	-181,92	-286,94	0,00	13,29	48,75
24	120,00	4319,45	2,0922	-1763,20	2895,84	2871,79	0,00	0,00	0,00	0,00	49,80	-184,92	-301,91	0,00	15,52	44,59
25	120,00	4439,45	2,1487	-1700,53	2998,15	2871,79	0,00	0,00	0,00	0,00	42,35	-186,52	-317,32	0,00	18,13	40,34
26	120,00	4559,45	2,2051	-1632,18	3096,76	2871,79	0,00	0,00	0,00	0,00	34,26	-186,67	-333,04	0,00	21,08	36,07
27	211,25	4725,08	2,2831	-1528,92	3226,21	1293,60	0,00	0,00	0,00	0,00	22,28	-184,44	-354,95	0,00	25,70	30,29
28	211,25	4936,33	2,3825	-1383,15	3378,98	1293,60	0,00	0,00	0,00	0,00	6,15	-177,40	-382,67	0,00	32,75	22,90
29	211,25	5147,58	2,4819	-1222,93	3516,53	1293,60	0,00	0,00	0,00	0,00	-10,21	-165,73	-409,37	0,00	40,65	16,11
30	211,25	5358,83	2,5813	-1049,85	3637,50	1293,60	0,00	0,00	0,00	0,00	-26,03	-149,63	-434,20	0,00	48,90	10,35
31	211,25	5570,08	2,6807	-865,62	3740,69	1293,60	0,00	0,00	0,00	0,00	-40,60	-129,44	-456,38	0,00	56,91	5,86
32	211,25	5781,33	2,7801	-672,06	3825,09	1293,60	0,00	0,00	0,00	0,00	-53,24	-105,68	-475,21	0,00	64,03	2,76
33	211,25	5992,58	2,8795	-471,07	3889,86	1293,60	0,00	0,00	0,00	0,00	-63,39	-78,96	-490,08	0,00	69,68	0,94
34	211,25	6203,83	2,9789	-264,65	3934,37	1293,60	0,00	0,00	0,00	0,00	-70,58	-50,02	-500,51	0,00	73,34	0,15
35	120,00	6369,45	3,0569	-100,33	3954,79	2871,79	0,00	0,00	0,00	0,00	-73,95	-26,29	-505,35	0,00	74,36	0,02
36	120,00	6489,45	3,1134	19,46	3961,57	2871,79	0,00	0,00	0,00	0,00	-75,07	-8,79	-506,97	0,00	74,74	0,00
37	120,00	6609,45	3,1698	139,44	3961,57	2871,79	0,00	0,00	0,00	0,00	-75,07	8,79	-506,97	0,00	74,74	0,00
38	120,00	6729,45	3,2263	259,23	3954,79	2871,79	0,00	0,00	0,00	0,00	-73,95	26,29	-505,35	0,00	74,36	-0,02
39	211,25	6895,08	3,3042	423,55	3934,37	1293,60	0,00	0,00	0,00	0,00	-70,58	50,02	-500,51	0,00	73,34	-0,15
40	211,25	7106,33	3,4037	629,97	3889,86	1293,60	0,00	0,00	0,00	0,00	-63,39	78,96	-490,08	0,00	69,68	-0,94
41	211,25	7317,58	3,5031	830,96	3825,09	1293,60	0,00	0,00	0,00	0,00	-53,24	105,68	-475,21	0,00	64,03	-2,76
42	211,25	7528,83	3,6025	1024,52	3740,69	1293,60	0,00	0,00	0,00	0,00	-40,60	129,44	-456,38	0,00	56,91	-5,86
43	211,25	7740,08	3,7019	1208,75	3637,50	1293,60	0,00	0,00	0,00	0,00	-26,03	149,63	-434,20	0,00	48,90	-10,35
44	211,25	7951,33	3,8013	1381,83	3516,53	1293,60	0,00	0,00	0,00	0,00	-10,21	165,73	-409,37	0,00	40,65	-16,11
45	211,25	8162,58	3,9007	1542,05	3378,98	1293,60	0,00	0,00	0,00	0,00	6,15	177,40	-382,67	0,00	32,75	-22,90
46	211,25	8373,83	4,0001	1687,82	3226,21	1293,60	0,00	0,00	0,00	0,00	22,28	184,44	-354,95	0,00	25,70	-30,29
47	120,00	8539,45	4,0781	1791,08	3096,76	2871,79	0,00	0,00	0,00	0,00	34,26	186,67	-333,04	0,00	21,08	-36,07
48	120,00	8659,45	4,1345	1859,43	2998,15	2871,79	0,00	0,00	0,00	0,00	42,35	186,52	-317,32	0,00	18,13	-40,34
49	120,00	8779,45	4,1910	1922,10	2895,84	2871,79	0,00	0,00	0,00	0,00	49,80	184,92	-301,91	0,00	15,52	-44,59
50	120,00	8899,45	4,2475	1978,90	2790,15	2871,79	0,00	0,00	0,00	0,00	56,48	181,92	-286,94	0,00	13,29	-48,75
51	211,25	9065,08	4,3254	2047,32	2639,36	1293,60	0,00	0,00	0,00	0,00	64,24	175,61	-267,26	0,00	10,75	-54,34
52	211,25	9276,33	4,4248	2117,19	2440,09	1293,60	0,00	0,00	0,00	0,00	71,27	164,25	-244,27	0,00	8,89	-59,64
53	211,25	9487,58	4,5242	2166,95	2234,87	1293,60	0,00	0,00	0,00	0,00	74,69	149,70	-224,19	0,00	8,15	-62,70
54	211,25	9698,83	4,6237	2196,09	2025,73	1293,60	0,00	0,00	0,00	0,00	74,19	132,60	-207,49	0,00	8,07	-63,27
55	211,25	9910,08	4,7231	2204,33	1814,73	1293,60	0,00	0,00	0,00	0,00	69,60	113,69	-194,48	0,00	8,15	-61,29
56	211,25	10121,33	4,8225	2191,59	1603,95	1293,60	0,00	0,00	0,00	0,00	60,93	93,84	-186,50	0,00	7,88	-56,92
57	211,25	10332,58	4,9219	2157,99	1395,48	1293,60	0,00	0,00	0,00	0,00	48,41	74,33	-184,25	0,00	6,85	-50,52
58	211,25	10543,83	5,0213	2103,86	1191,37	1293,60	0,00	0,00	0,00	0,00	32,56	56,44	-187,39	0,00	4,76	-42,64
59	120,00	10709,45	5,0992	2047,42	1035,71	2871,79	0,00	0,00	0,00	0,00	18,28	44,31	-193,25	0,00	2,28	-35,81
60	120,00	10829,45	5,1557	1999,03	925,91	2871,79	0,00	0,00	0,00	0,00	7,20	36,87	-199,11	0,00	0,12	-30,90
61	120,00	10949,45	5,2239	1944,21	819,19	2871,79	444,00	0,00	0,00	0,00	-4,17	27,92	-205,76	88,45	-2,34	-26,10
62	120,00	11069,45	5,3039	1881,35	717,01	2871,79	444,00	0,00	0,00	0,00	-14,38	18,76	-213,79	76,75	-5,16	-21,52
63	181,67	11220,29	5,4045	1791,09	596,24	1293,60	672,17	0,00	0,00	0,00	-25,42	12,27	-225,48	67,71	-9,15	-16,18
64	181,67	11401,95	5,5256	1666,98	463,73	1293,60	672,17	0,00	0,00	0,00	-35,70	10,58	-240,82	65,61	-14,20	-10,80
65	181,67	11583,62	5,6467	1527,77	347,19	1293,60	672,17	0,00	0,00	0,00	-42,33	14,42	-255,76	71,61	-19,15	-6,65
66	181,67	11765,29	5,7678	1375,50	248,31	1293,60	672,17	0,00	0,00	0,00	-44,97	22,25	-268,78	82,9		

- $N_{Fi}$  ... the normal force in the  $i^{\text{th}}$  segment caused by  $F$  only [kN],
- $V_{Fi}$  ... the shifting force in the  $i^{\text{th}}$  caused by  $F$  only [kN].

Each row in the column gives the results for the  $i$  row. Yellow rows are the rows where the segments overlap in joints. Colourless rows are the rows with simple sections. The red row is the row where the ultimate state of support is reached.

- **Graph shape** The support's shape and the point where  $F$  is applied (if specified). See Fig. 14. If any point on the arch's central line is clicked, the system displays  $x$  and  $y$  coordinates of that point in mm. The shape is important for validation of correctness of parameters specified in the support geometry.
- **Graph  $N_{ci}$**  The normal force along the line of the developed central line for the closed support that starts in  $D$ . The distance of the central line  $L_i$  is calculated clockwise from  $D$ . See Fig. 15. For each  $i$ , the distance  $L_i$  is calculated in **Final data result** (see the bar in Fig. 9). If the sign is plus, the normal force is tensile one. If the sign is minus, the normal force is compressive.
- **Graph Arch  $N_{ci}$**  The normal force along the central line of the closed support. See Fig. 16.
- **Graph  $M_{ci}$**  The bending moment along central line of the closed support that is developed into a line. See Fig. 17. The positive bending moment results in tension in inner fibres and compression in outer fibres of the closed steel support.
- **Graph Arch  $M_{ci}$**  The bending moment along the central line of the closed support. See Fig. 18.

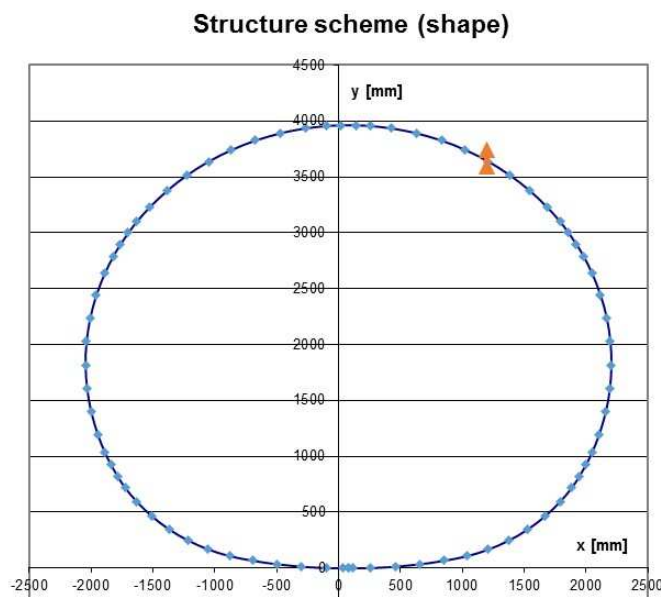


Fig. 14 The support's shape and the point where  $F$  is applied.



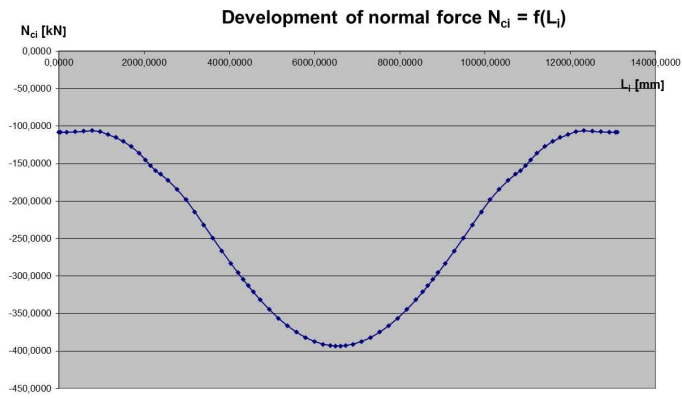


Fig. 15 Normal force  $N_{ci}$  [kN] along the central line of the closed support that is developed into a line.

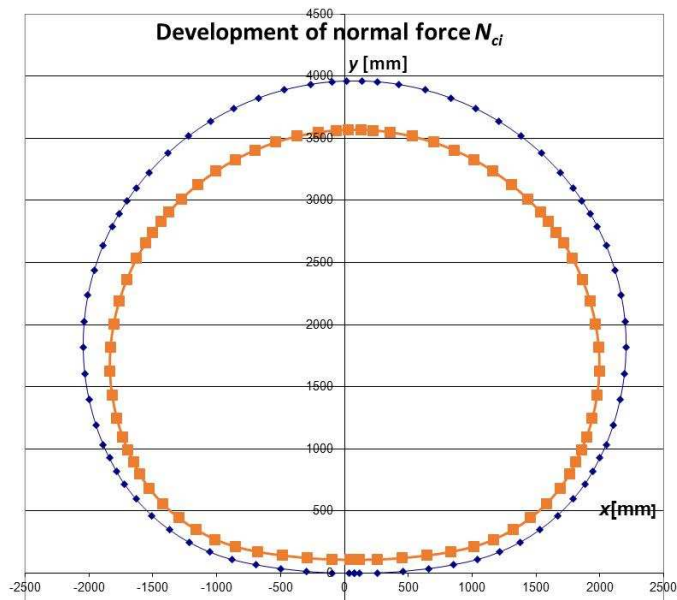


Fig. 16. The normal force  $N_{ci}$  along the central line of the closed support.

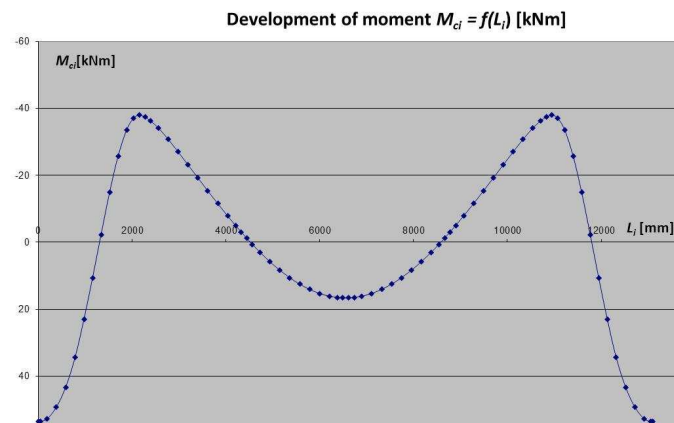


Fig. 17 The bending moment  $M_{ci}$  [kNm] along the central line of the closed support that is developed into a line.



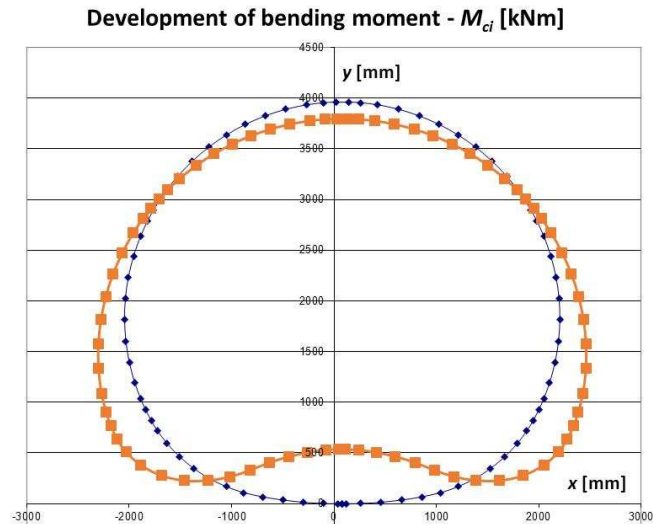


Fig. 18. The bending moment  $M_{ci}$  [kNm] along the central line of the arch.

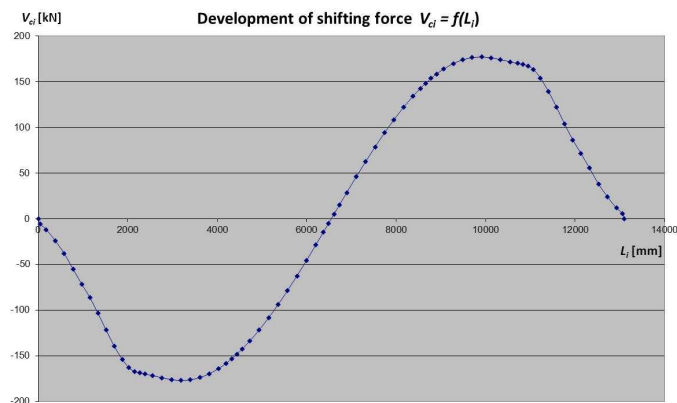


Fig. 19 The shifting force  $V_{ci}$  [kN] along the central line of the closed support that is developed into a line.

- **Graph  $V_{ci}$**  The shifting force along the central line of the closed support that is developed into a line. See Fig. 19.
- **Graph Arch  $V_{ci}$**  The shifting force along the central line of the arch.
- **Graph  $u_i$**  The horizontal shift  $u_i$  along the central line of the closed support. See Fig. 20. The sign plus for  $u_i$  means the positive direction of the  $x$  axis.  $u_i$  and  $v_i$  are calculated for the load that corresponds to the specified load capacity of the support.
- **Graph Arch  $u_i$**  The vertical shift  $u_i$  along the central line of the closed support. See Fig. 21. The central line of the closed support is highlighted in blue. If the orange connection line of points is out of the contour, it diverges. If it is inside the contour, it converges.
- **Graph  $v_i$**  The vertical shift  $v_i$  along the central line of the closed support. See Fig. 22. The sign plus for  $v_i$  corresponds to the positive direction of the  $y$  axis.
- **Graph Arch  $v_i$**  The vertical shift  $v_i$  along the central line of the closed support. See Fig. 23. The shift of the orange connection line against the blue line shows movement of the closed support against the direction of the  $y$  axis.

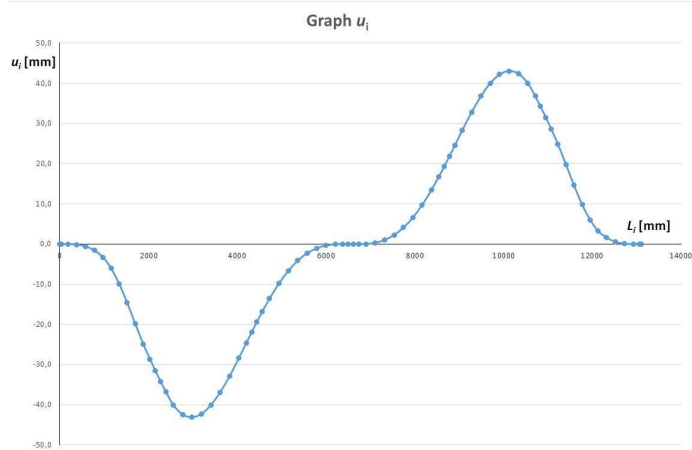


Fig. 20. Shift  $u_i$  [mm] along the central line of the closed support that is developed into a line.

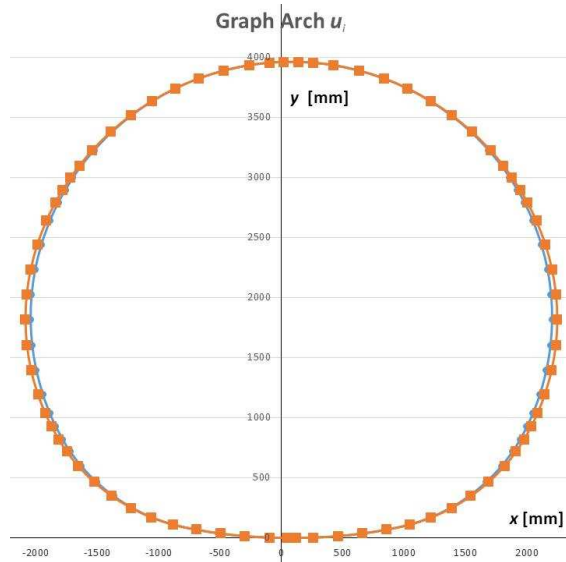


Fig. 21. The horizontal shift  $u_i$  along the central line of the closed support.

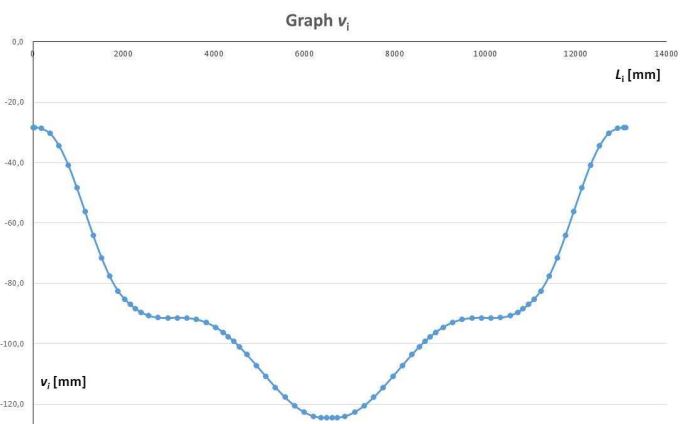


Fig. 22. Shift  $v_i$  [mm] along the central line of the closed support that is developed into a line.

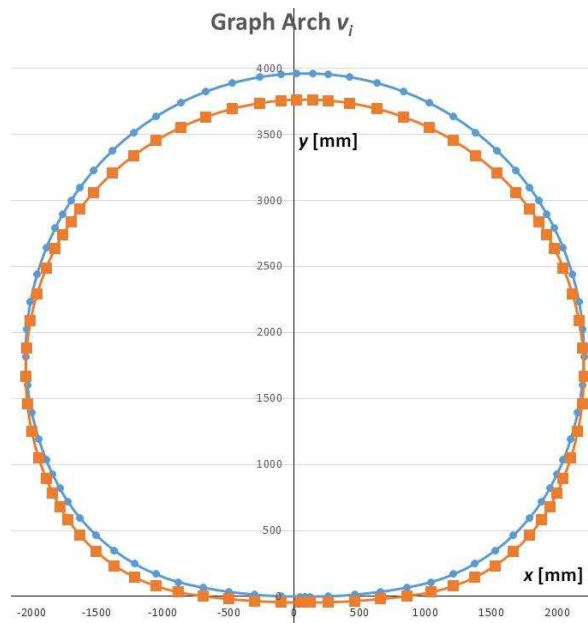


Fig. 23. The vertical shift  $v_i$  along the central line of the closed support.

- **Graph  $q_{p,i}$**  The passive forces  $q_{p,i}$  along the central line of the closed support. See Fig. 24. These are compressive forces and are normal to the central line.
- **Graph Arch  $q_{p,i}$**  The passive load  $q_{p,i}$  in the closed support. See Fig. 25.

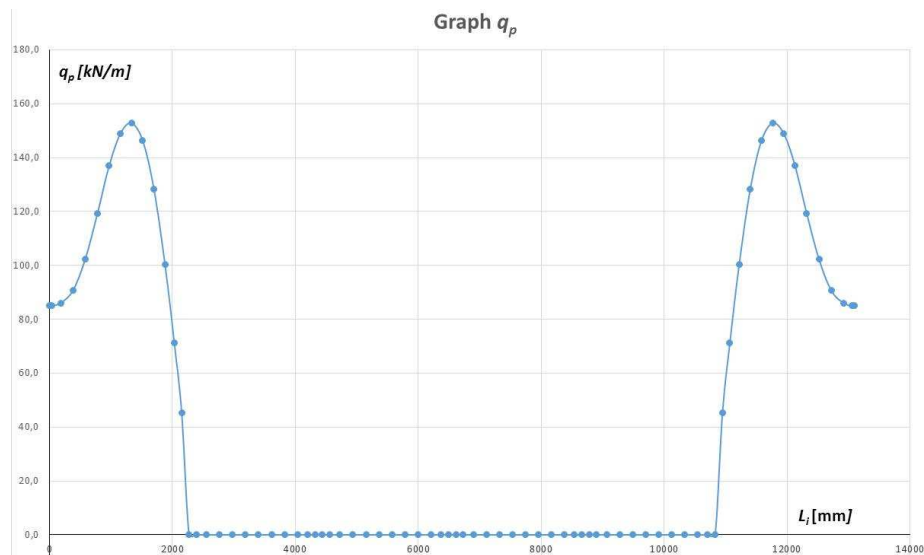


Fig. 24. The passive load  $q_{p,i}$  [kNm<sup>-1</sup>] along the central line of the closed support that is developed into a line.

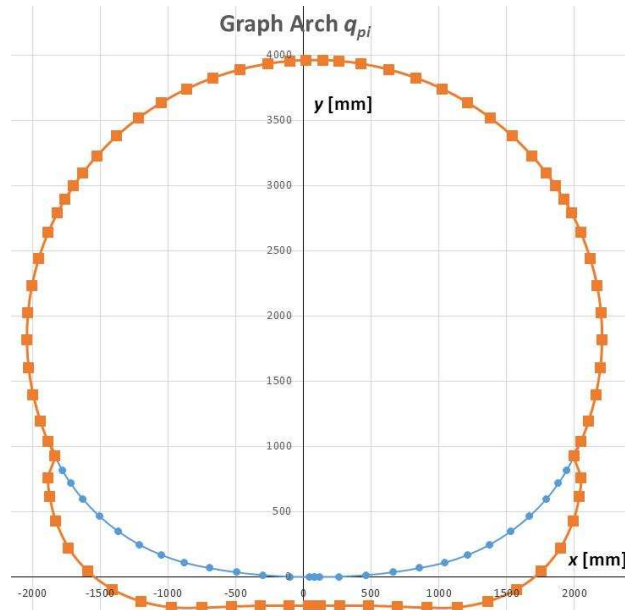


Fig. 25. The passive load  $q_{pi}$  in the closed support.

- Graph eps-q** The loading capacity of the support that depends on  $\epsilon$ — see Fig 26. The graph is displayed if marked in the bar. The load capacity is displayed for  $\epsilon$  changing from 0 to 2 with the 0.2 step for  $F=0$ . For calculation, check the checkbox **Graph  $\epsilon / q$**  in the right pane of the input form and specify the step. A warning message (shown in Fig. 27) appears when enter values  $D_{ci}$  are not symmetrical, and the values  $D_{ci}$  are modified automatically. The calculation is performed when the user clicks OK.

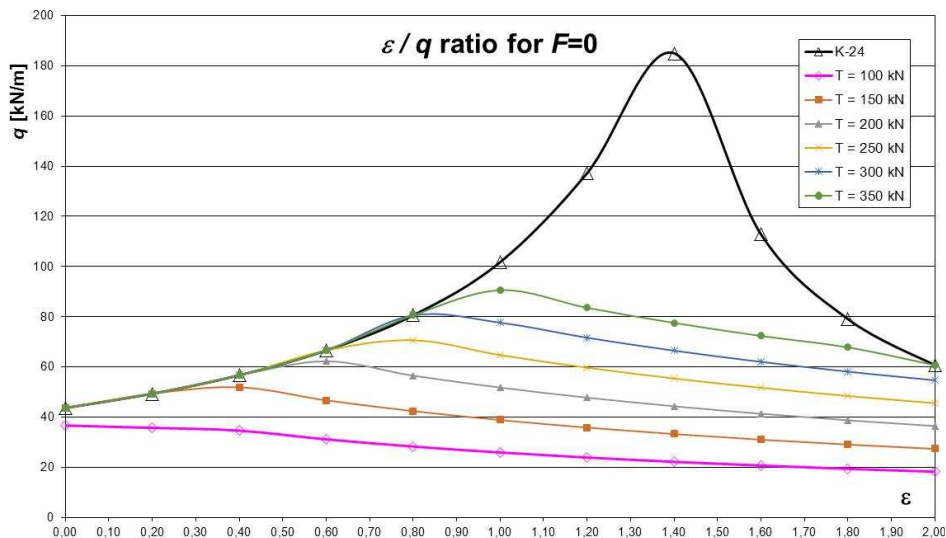


Fig. 26 The load capacity of the support  $q = f(\epsilon)$  (the MP 1/K24/H60U section) for the specification in Fig. 7.

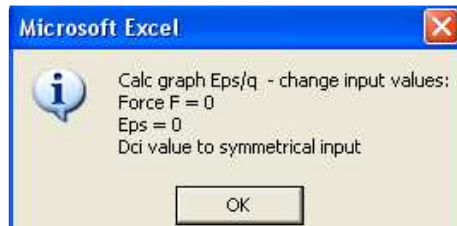


Fig. 27. Notice of the change of the values of the  $D_{ci}$ .

- **Eps-q** The table with the values of loading capacity  $q$  for variol  $\epsilon$ . For calculation, check the checkbox at **Graph  $\epsilon/q$**  in the right pane of the input form and specify the step.
- **Graph eps-q<sub>h</sub>** The loading capacity,  $q_h$  of the support that depends on  $\epsilon$  see Fig 28. The graph is displayed if marked in the bar. The load capacity is displayed for  $\epsilon$  changing from 0 to 2 with the 0.2 step for  $F=0$ . For calculation, check the checkbox at **Graph  $\epsilon/q$**  in the right pane of the input form and specify the step.

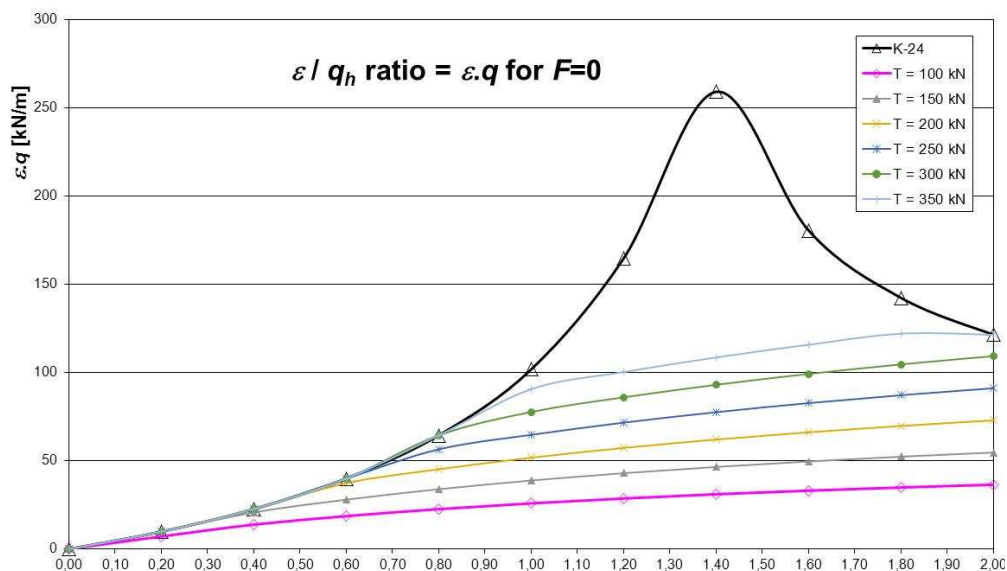


Fig. 28 The load capacity of the support  $q_h = f(\epsilon)$  (the MP 1/K24/H60U section) for the specification in Fig. 7.

Fig. 26 and 28 indicate clearly that the load capacity of the support,  $q$  and  $q_h$ , changes in a really wide interval. It depends, in addition to the support shape and weight level (and steel grade), considerably on the load which is characterised by  $\epsilon$  and on the slippage resistance. It also depends on yield of the neighbouring environment. The yield values in the figure correspond to the specification in Fig. 7, the only change being that  $\epsilon$  ranges from 0 to 2 and the step is 0.2. In Fig. 29 there are the values calculated for different  $\epsilon$ . The difference against Fig. 7 is that  $D_c = 3000 \text{ kNm}^{-2}$  is considered for the entire perimeter of the support. See the specification in 30.

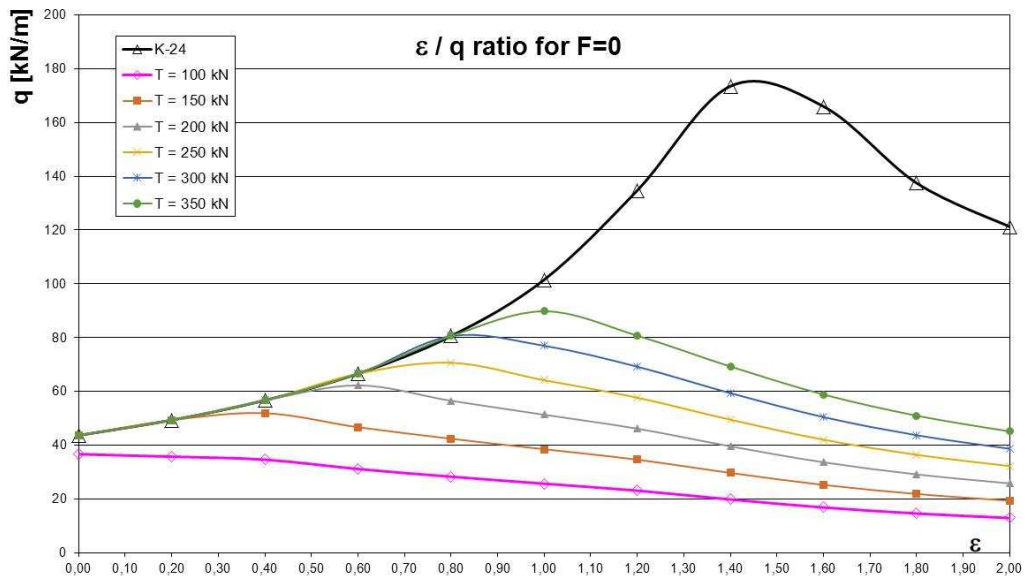


Fig. 29 The load capacity of the support  $q = f(\epsilon)$  (the MP 1/K24/H60U section) for the specification in Fig. 30.

Arch length [mm]	Radius [mm]	Overlapping [mm]	Index Dci [kN/m <sup>2</sup> ]
L1: 800	R1: 3880	O 1-2: 0	Dc1: 3000
L2: 1570	R2: 1500	O 2-3: 480	Dc2: 3000
L3: 2650	R3: 2125	O 3-4: 480	Dc3: 3000
L4: 2650	R4: 2125	O 4-5: 480	Dc4: 3000
L5: 2650	R5: 2125	O 5-6: 480	Dc5: 3000
L6: 2650	R6: 2125	O 6-7: 480	Dc6: 3000
L7: 1570	R7: 1500	O 7-8: 0	Dc7: 3000
L8: 800	R8: 3880	O 8-1: 480	Dc8: 3000

Fig. 30. Specification for calculation of the load capacity of a support pursuant to Fig. 29

### 3.3 Using the software for calculation of the support's load capacity – some examples

The closed support MP1 consisted of curved segments only that had one or two radii of curvature. In some cases, the segments in the support may have a straight section. For instance, see the support RP4 in Fig. 31. This is, in fact, an open arch support with an anti-vault. The anti-vault consists of three parts, while the arch support consists of five parts. The anti-vault is modelled using 4 segments that are defined by the lengths  $L_1$ ,  $L_2$ ,  $L_{14}$  and  $L_{15}$ , the radii of curvature and the overlapping with the neighbouring segments. The lowest segment was divided into two parts. This means, the overlapping between the 1<sup>st</sup> and 15<sup>th</sup> segment is zero. The arch support is characterised by the segments, the lengths of which are  $L_4$ ,  $L_5$ ,  $L_6$ ,  $L_7$ ,  $L_8$ ,  $L_9$ ,  $L_{10}$ ,  $L_{11}$  and  $L_{12}$ . The segments  $L_4$  and  $L_5$  were created by division of the only one segment with two different radii of curvature where the large value  $R_4$  ( $R_{12}$ ) simulates the straight portion of the segment. The overlapping for the segments 4 and 5 is zero. The same situation exists for the segments 11 and 12. The segments with the lengths  $L_6$  and  $L_7$ , and those with the lengths  $L_8$  and  $L_9$ , were created by dividing one segment into two segments. The reason was a different radius of

curvature. The anti-vault is connected with the arch support through the segments 3 and 13. Their length is short and their radius of curvature is small. They also simulate a hinge joint. The first calculation without the hinges identifies the position of the segments 3 and 13 where the hinges will be placed (see **Final data**). The hinges are situated in points 17 and 84. Calculation results are shown in Fig. 31.

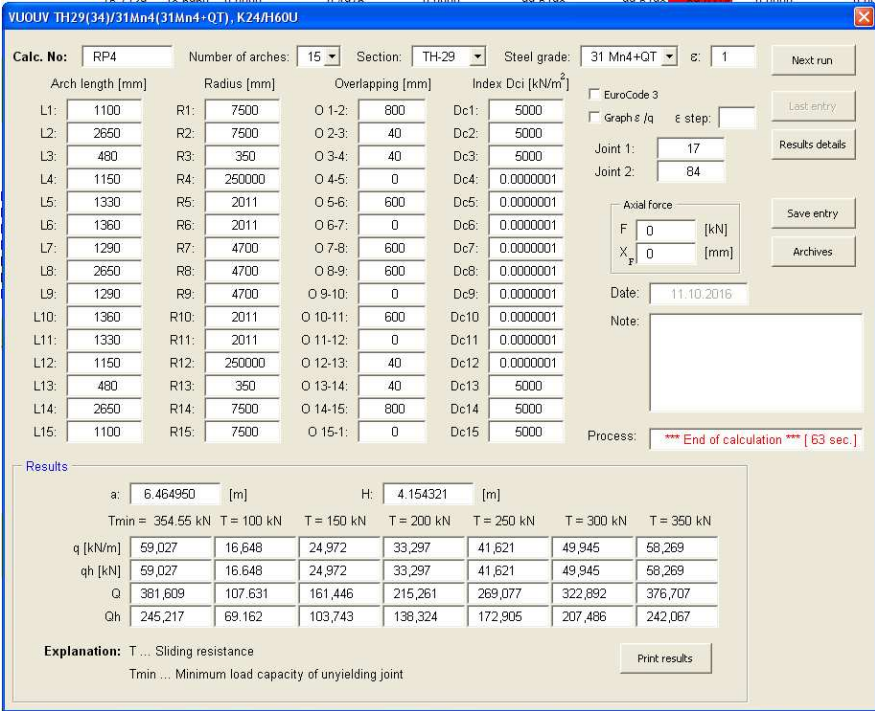


Fig. 31. The specification and calculation of the load capacity for the RP4 support.

The difference between Fig. 33 and Fig. 31 is that  $D_{c,i}$  in points where it is not zero is 10 times lower. This is evident for the load capacity that is lower in Fig. 33 than in Fig. 31, the evidence being the progress of internal forces, the shift and passive forces. Compare please Fig. 32 and Fig. 34. If the base is more yielding, then the support segments are pressed into the base soil along a bigger length, reducing thus the load capacity of the segments and, in turn, of the entire support.

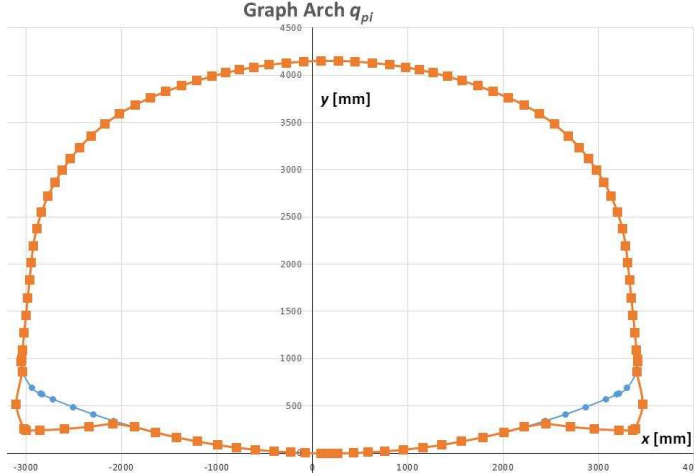


Fig. 32. Passive forces in the calculation pursuant to Fig. 30



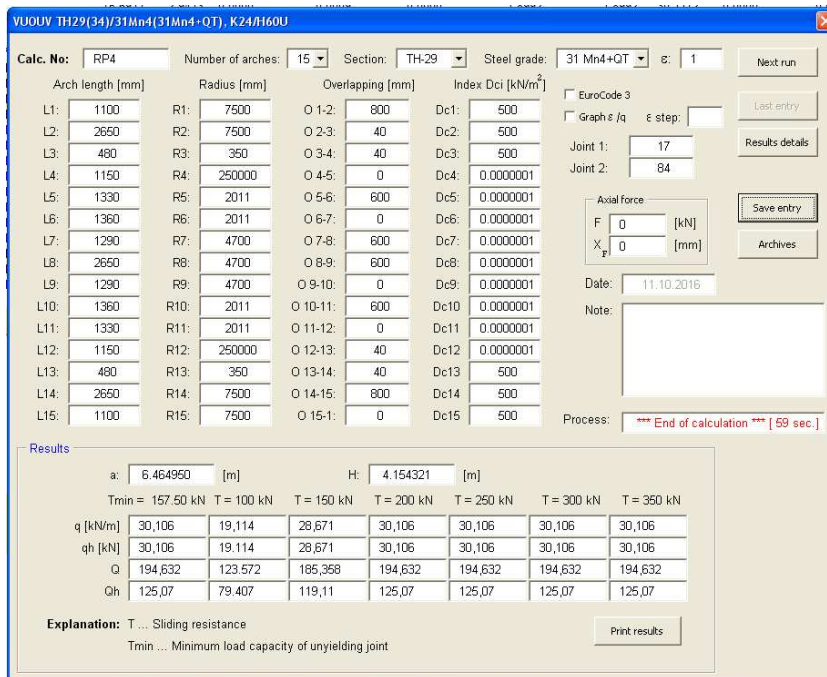


Fig. 33. The specification and calculation of the load capacity for the RP4 support.

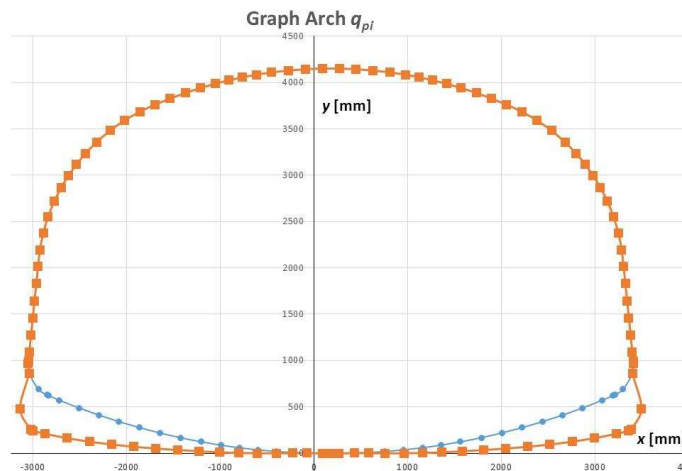


Fig. 34. Passive forces in the calculation pursuant to Fig. 32

Fig. 35 specifies the calculation of the load capacity for the MP4 support. This support consists of 7 segments, each of them being 2,650 mm long. Two segments have two different radii of curvature. Consequently, they were modelled as two separate segments with zero overlap (the segments 2 and 3 and the segments 8 and 9). For purposes of calculation, the lowest segment will be divided into two pieces (the segments 1 and 10). This means that the support was modelled as a 10 segment support. It was loaded by the continuous load from above and from the sides where  $\epsilon = 1$ . The yield of the surrounding environment was modelled for the entire perimeter using  $D_c = 10000 \text{ kNm}^{-2}$ . Passive forces (see Fig. 36) developed in arches on the base soil, though they were specified for the entire perimeter of the support. Most passive forces between the rock



and support were strength forces - this situation cannot occur. As mentioned above, such forces are excluded from the calculation.

The specification for Fig. 37 is same as for Fig. 35. The difference is only in  $D_c$  that is  $D_c=1000 \text{ kNm}^{-2}$  only on the right side. The exception is the last arch (10) where the original value was kept. This change in the specification will reduce the load capacity of the support which is clear from the comparison of Fig. 35 and Fig. 37. Distribution of passive forces will change: they will be the highest in locations where the solid rock or rock fill is most yielding. Regarding the shift of the support, the situation is different (see Fig. 39).

The passive forces and shift in the support load depend very much on the yield of the surrounding rock. In particular, they depend on the asymmetric specification for the arches that are placed at the lowest level. If the shift is more than 10 % of the maximum width of the work  $a$ , the warning will appear. See Fig. 40.

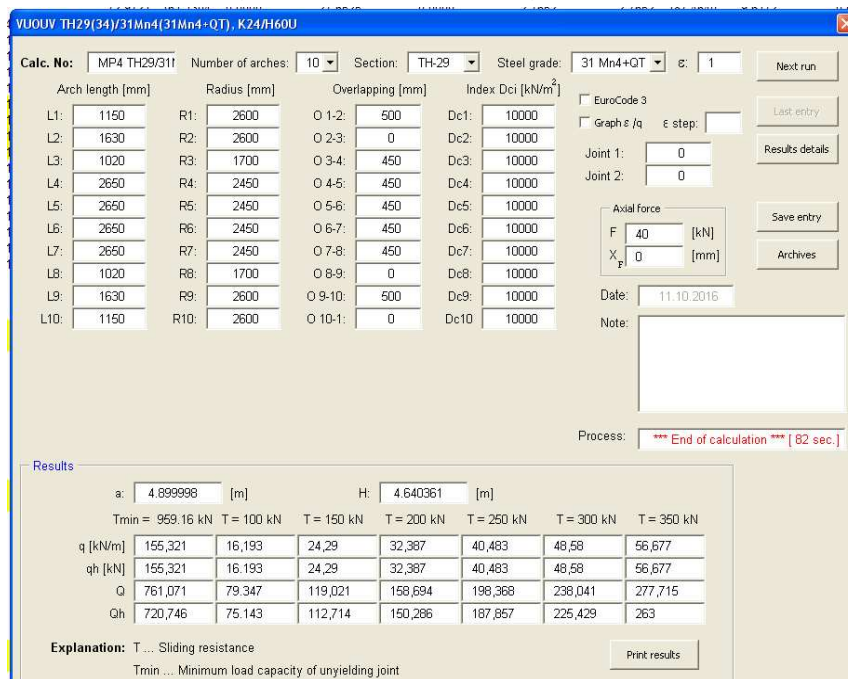


Fig. 35. The specification and calculation of the load capacity for the MP4 support.

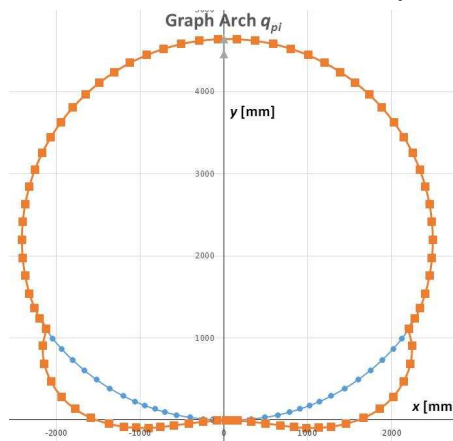


Fig. 36. The passive load  $q_{p,i}$  in the closed support MP4 with the specification in Fig. 34.

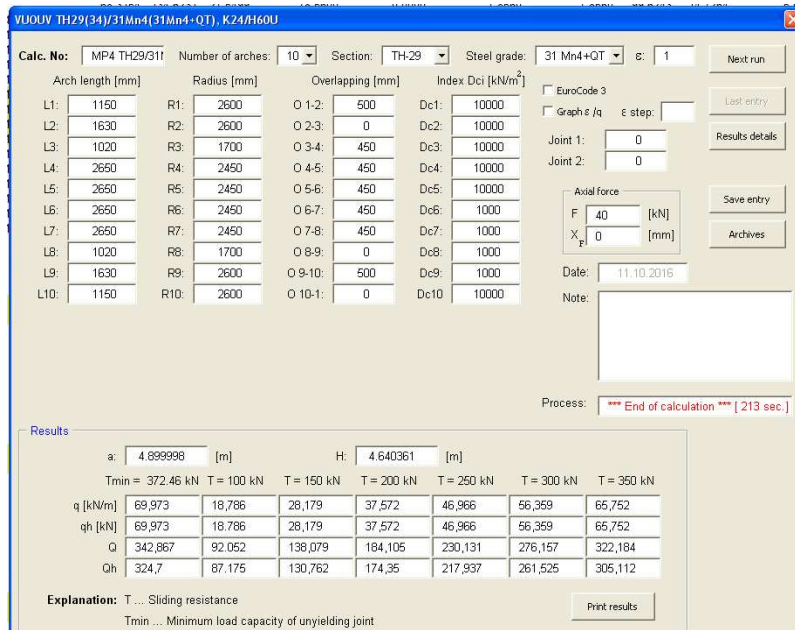


Fig. 37. The specification and calculation of the load capacity for the MP4 support for different  $D_{ci}$ .

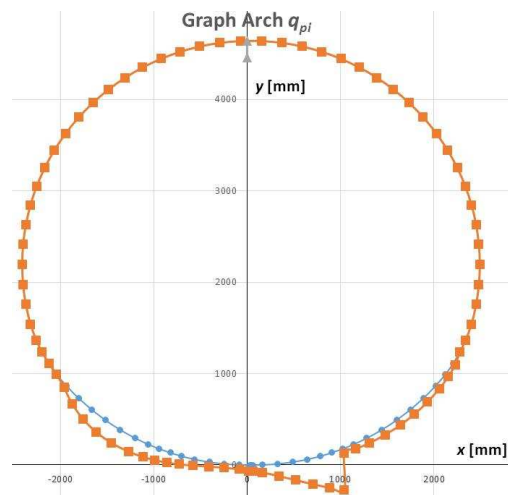


Fig. 38. The passive load  $q_{p,i}$  in the closed support MP4 with the specification in Fig. 36.

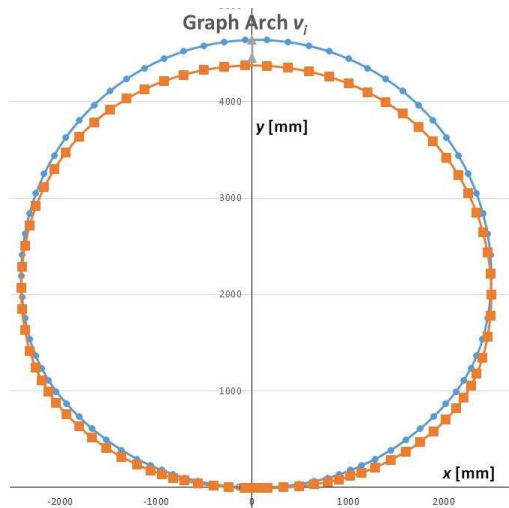


Fig. 39. The vertical shift  $v_i$  in the closed support MP4 with the specification in Fig. 37.

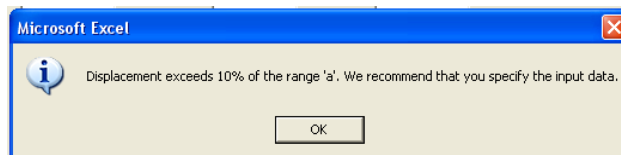


Fig. 40. The warning message for a too high shift.

When you press **Print results**, the data and calculation results are displayed for the specified data. See Fig. 41.

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*Calculating the load capacity of closed steel supports in galleries  
(VÚOUV TH29(TH34)/31Mn4(31Mn4+QT) and K24/H60U  
prepared for the Premegovnik Mine (Velenje) in Slovenia*

Calculation No.: MP4 TH29/31Mn4+QT      Date: 11.10.2016

**Entries:**  
Number of arches: 10    Section: TH-29    Steel grade: 31 Mn4+QT    Eps: 1    EuroCode 3: No  
L1: 1150    L2: 1630    L3: 1020    L4: 2650    L5: 2650    L6: 2650  
L7: 2650    L8: 1020    L9: 1630    L10: 1150  
R1: 2600    R2: 2600    R3: 1700    R4: 2450    R5: 2450    R6: 2450  
R7: 2450    R8: 1700    R9: 2600    R10: 2600  
D1: 500    D2: 0    D3: 450    D4: 450    D5: 450    D6: 450  
D7: 450    D8: 0    D9: 500    D10-D1: 0  
Dc1: 10000    Dc2: 10000    Dc3: 10000    Dc4: 10000    Dc5: 10000    Dc6: 1000  
Dc7: 1000    Dc8: 1000    Dc9: 1000    Dc10: 10000  
Joint 1 = 0    Joint 2 = 0    Axial force: F = 40 [kN]    X = 0 [mm]

**Results:**  
Values for Tmin = 372,46 kN  
q=69,973 [kN/m]    qh=69,973 [kN]    Q=342,867 [kN]    Qh=324,7 [kN]  
Values for T = 100 [kN]  
**q=18,786 [kN/m]    qh=18,786 [kN]    Q=92,052 [kN]    Qh=87,175 [kN]**  
Values for T = 150 [kN]  
q=28,179 [kN/m]    qh=28,179 [kN]    Q=138,079 [kN]    Qh=130,762 [kN]  
Values for T = 200 [kN]  
q=37,572 [kN/m]    qh=37,572 [kN]    Q=184,105 [kN]    Qh=174,35 [kN]  
Values for T = 250 [kN]  
q=46,966 [kN/m]    qh=46,966 [kN]    Q=230,131 [kN]    Qh=217,937 [kN]  
Values for T = 300 [kN]  
q=56,359 [kN/m]    qh=56,359 [kN]    Q=276,157 [kN]    Qh=261,525 [kN]  
Values for T = 350 [kN]  
q=65,752 [kN/m]    qh=65,752 [kN]    Q=322,184 [kN]    Qh=305,112 [kN]

Fig. 41 Print results.

### 3.4 Using the archived data

Calculation of the load capacity of steel supports is rather fast and it does not seem to be necessary to archive extensive files of calculations and results. It is more reasonable to save the input data which provide same results, unless unauthorised changes are introduced into the software application. It is easy to archive the input data. The archiving process is described above. To load the stored data, press the **Archives** button in the input and output form sheets.

To repeat the calculation with the saved input data, click the **Last entry** button first. To load the saved input data, click **Archives**. The necessary values can be obtained using, for instance,

- the section No.
- the section
- the number of curves (segments)
- date of performance from – to.

after you click **Search**. The list from which the data are selected is thus reduced considerably. Click the entry which should be calculated again. After you click **OK**, the values will be loaded for the re-calculation. Then continue as usually. This means, click **Run**.

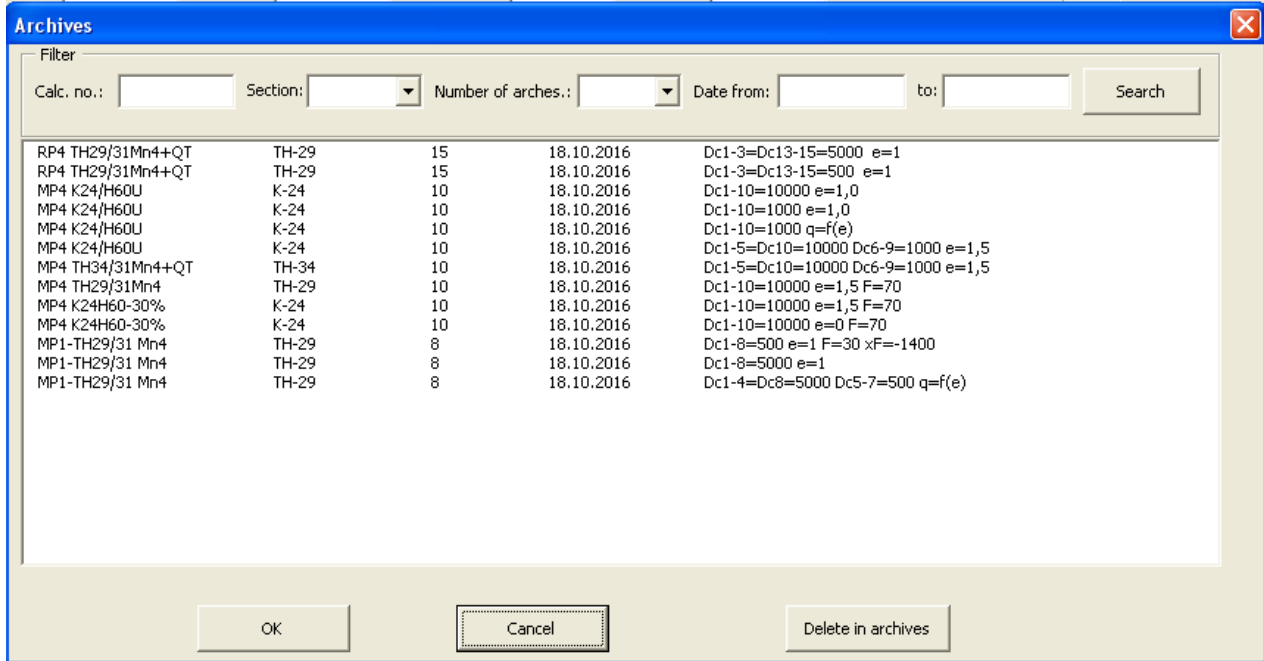


Fig. 42 Searching the input data in the archives

After you click **Cancel**, the input form sheet or the form sheet with final results will be displayed. The highlighted data in the archives will be deleted once you click **Delete on archives**.

## 4. Conclusion

The software *Calculation of load capacity of closed steel supports – VÚOUV TH29(TH34)/31Mn4(31Mn4+QT) and K24/H60U* can be used for various tasks which need to be addressed during development, design, and assessment of the closed steel supports in long mining works and underground workings with the specified supports installed in different conditions. It is possible to include into the calculation the vertical axial force which is applied in the specified point. The vertical axial force influences the load capacity of the support which is defined by the permitted continuous load applied onto the support from top or, from sides of the mining work/underground working. The support is also subject to passive forces induced mainly in base surface (and in other points along the perimeter) where the reason for such forces is interaction between the support and the neighbouring rock environment or fill rock. The Winkler's model was used to solve the interaction. It is recommended to carry out experiments and to validate the yielding coefficient of rock environment (including the fill rock) that is considered in the Winkler's model.

In case of non-yielding supports, the limiting factor for reaching the ultimate state in any point of the support is the plastic moments and the limit normal force. Those values are obtained from known properties of the steel, cross-section characteristics of the sections and results of the bending tests in straight rods resulting mostly from the project TA ČR: TA01010838: research and development of controlled rolling production of steel mine supports and bending tests performed in March 2016 [10].

Each support which has been designed as a yielding support where the segments are connected by U-bolts or screwed joints can behave as a yielding or non-yielding support. There are several reasons to this and most of them have been mentioned above. One of the reasons is, clearly, the resistance against slippage of the connected segments. It is assumed in the calculation that the slippage resistance is known. That value should be determined and validated experimentally for each section made from the specified material and for each fastener and U-bolt/screwed joint. It is not recommended and not advisable to use a slippage resistance if it has not been confirmed by experiments.

The software *Calculation of load capacity of closed steel supports – VÚOUV TH29(TH34)/31Mn4(31Mn4+QT) and K24/H6* must not be manipulated unless consent is provided by the software developers. If anything is not clear in the software use or if problems are faced, contact please the software developers who will be also grateful for any experience, suggestions or knowledge obtained during the use of the software.

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